Commodity Exporters, Heterogeneous Importers, and the Terms of Trade*

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Abstract

How important are shocks to the terms of trade relative to TFP shocks as a source of consumption volatility in commodity-exporting economies when firms are heterogeneous? In light of mounting evidence of heterogeneity in firm-level trade adjustment, we develop an analytical framework that nests a benchmark Small-Open Economy International Real Business Cycle (SOE-IRBC) model, a tractable general equilibrium version of Gopinath & Neiman (2014), and several frameworks in between. The analysis yields three key theoretical results. First, the equilibria of the models are the fixed point of a single equation in the economy's trade openness, which coincides with the imports-to-consumption ratio. Second, the differences between the models are captured by two elasticities that relate changes in key aggregate variables to changes in trade openness. Finally, the relative importance of terms of trade shocks depends on one general equilibrium elasticity, which we call the terms-of-trade elasticity, independent of assumptions on market structure, returns to scale, and selection into importing. As the terms-of-trade elasticity depends on equilibrium trade openness, we find that the different models predict virtually the same relative importance of shocks to the terms of trade shocks when calibrated to match the same level of trade openness. Our results suggest that matching key micro-moment of heterogeneous trade adjustment across firms does not change the relative importance of terms-of-trade shocks in generating aggregate fluctuations once trade openness is accounted for.

JEL codes: E13, E32, F32, F41 and F44
Keywords: Business cycles, Trade adjustment, Terms-of-trade and Commodity prices

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1 Introduction

Emerging economies are characterized by substantial volatility in final consumption. Through the lens of International Real Business Cycles (IRBC) models, a large literature investigates which shocks cause this volatility and how important unobserved shocks, such as sectoral total factor productivity (TFP) shocks, are relative to observable shocks, like for example terms-of-trade shocks. Unquestionably, the more models reduce their reliance on unobserved shocks, the better they become (e.g. Abramovitz (1956); Cochrane (1994)). Although studies differ in the exact share attributed to different shocks, most studies agree that substantial sectoral TFP shocks are needed to replicate the volatility observed in the data.

At the same time, there is mounting evidence of heterogeneous trade adjustment across importing firms (e.g. Amiti & Konings (2007), Goldberg et al. (2010), Gopinath & Neiman (2014) and Halpern et al. (2015)). Aggregate imports adjust because large continuing importers adjust their firm-level imports and because small firms start and stop importing. Whereas small firms predominantly change the set of imported varieties, large importers also change the imported amount of each variety. Importantly, this literature stresses that because bigger firms are more exposed to international shocks and adjust on multiple margins, terms-of-trade shocks can induce considerable endogenous movements in aggregate productivity through reallocation across firms. Since IRBC models focus on equilibria with perfectly competitive homogenous firms, they cannot account for heterogeneous trade adjustment across firms. As a consequence, these models potentially miss such endogenous aggregate productivity movements that could lower the reliance on exogenous TFP shocks when explaining consumption volatility.

In this paper, we study whether models that can generate heterogeneous trade adjustment also predict that shocks to the terms of trade are relatively more important than models that do not. To do so, we develop a framework that inserts the partial equilibrium model proposed by Gopinath & Neiman (2014), which generates heterogeneous trade adjustment across firms, into a benchmark Small Open Economy IRBC (SOE-IRBC) model of a commodity-exporting economy à la Mendoza (1995). In this way, our framework nests a frictionless benchmark SOE-IRBC model with representative producers, a general equilibrium version of the heterogeneous trade adjustment model, and other models in between.

The benchmark SOE-IRBC model is composed of a manufacturing sector and a final good sector with representative producers that compete under perfect competition. Manufacturing firms produce according to a constant returns-to-scale technology that combines labor and an input bundle consisting of domestic and foreign intermediate inputs. The final good is produced by combining labor and output from the manufacturing sector through a constant returns-to-scale technology as well. Finally, there is the commodity sector which is modeled as a time-varying endowment that affects domestic households' disposable income through the budget constraint.

To understand the contribution of each additional friction present in the heterogeneous trade adjustment model relative to the SOE-IRBC benchmark model, we move from the latter to the former in three steps. First, we consider the role of monopolistic competition in the manufacturing sector, which distorts the relative price of domestic and foreign intermediate inputs. Second, we add increasing returns to scale to importing. With this technology manufacturers trade-off the benefits from additional intermediate input varieties stemming from the love-for-variety aggregator on the imported intermediate input bundle with paying a constant fixed cost per imported variety in terms of domestic labor.[1] Finally, to capture heterogeneity in firm-level trade adjustment, we introduce heterogeneity in firm-level productivity and allow firms to endogenously select into and out of importing. In this way, the optimal number of intermediate input varieties also varies across firms of different sizes.

Our analysis yields three theoretical results. First, across all models considered, the non-linear zero-debt equilibrium is represented by one non-linear equation in one endogenous aggregate variable, which we call the “trade openness”. While trade openness is a function of the set of a set of structural parameters and its exact analytical expression varies across the models, it always represents the imports-to-final consumption ratio of the economy. Therefore, it captures how reliant the economy is on imported intermediate inputs to produce final consumption.

Second, up to first-order approximation, the equilibrium process of aggregate consumption is described by an equation in which only the elasticities attached to the exogenous shocks are model-dependent. For instance, in financial autarky, the response of final consumption to a terms-of-trade shock is summarized by one elasticity, which we refer to as the terms-of-trade elasticity. We show that the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining the variance of final consumption is summarized by an expression that only depends on intermediate input shares and the terms-of-trade elasticity. Moreover, the share explained by terms-of-trade shocks is rising in the terms-of-trade elasticity. Therefore, comparing the relative importance of terms-of-trade and TFP shocks in driving consumption volatility across models can be done by solely looking at the terms-of-trade elasticity.

The final theoretical result is that the terms-of-trade elasticity can be decomposed into two intuitive parts. The first part is simply the product technology parameters, that is the intermediate input shares in services and manufacturing, and the steady-state trade openness, which differs across the models. The more production relies on intermediate inputs, and the more those intermediate inputs are sourced from abroad, the more shocks to the terms of trade matter in explaining consumption volatility. As we deviate from the SOE-IRBC benchmark model and add frictions, two competing forces change the terms-of-trade elasticity relative to the one in the SOE-IRBC benchmark model. On the one hand, domestic distortions increase the incentives for manufacturing producers to import intermediate inputs, which increases equilibrium trade openness and exposure to external shocks. On the other hand, domestic distortions in the manufacturing sector also change the allocation of labor to the final goods sector, which can either

[1] Gopinath & Neiman (2014) shows that this friction is essential to capture import adjustment through the changing the amount imported of a given set of intermediate input varieties and the through the changes in the set of imported intermediate input varieties

increase or reduce the sensitivity of the labor allocation to final the goods sector to exogenous shocks.

Before quantitatively evaluating the models, we show that these results are robust to changing some of the simplifying assumptions we make to derive the results. For instance, accounting for endogenous adjustment of the amount of labor that is supplied by consumers changes the relative importance of terms-of-trade shocks to productivity shocks only by changing the terms-of-trade elasticity. At the same time, the terms-of-trade elasticity in perfect competition remains equal to the product of the intermediate input shares and equilibrium trade openness. If we allow consumers to share risk internationally, the equilibrium process for consumption changes. However, in the situation when the exogenous shocks approach random walks, the share explained in the growth rate of consumption by terms-of-trade shocks relative to productivity shocks remains pinned down by the same expression as in financial autarky. Hence, the expression remains a useful limiting result and this is true for most popular international market structures, including non-state contingent local and foreign currency bonds and segmented financial markets as in Itskhoki & Mukhin (2021).

To quantitatively evaluate the models, we calibrate the model with heterogeneous trade adjustment using macro data and firm-level trade data of Colombia and Chile. We show that the model with heterogeneous trade adjustment captures the main stylized facts of heterogeneous trade adjustment across firms. First, importers are larger both in sales and employment than non-importers. Second, the distribution of imports per firm follows a Generalized Pareto distribution and is therefore highly skewed, with a few firms importing large volumes and many firms importing small amounts. Third, larger importers import a more diversified set of goods, rarely stop importing altogether, and mostly adjust on the intensive margin while smaller importers adjust on the extensive margin. Fourth, larger importers adjust their imports mostly on the sub-intensive margin, while smaller firms adjust on the sub-extensive margin and we provide an expression for the relevance of the sub-intensive margin across the firm size distribution and show that it closely matches its empirical counterpart. Finally, through the reallocation of resources across importers of different sizes and through the entry and exit of firms into and out of importing, the complete model generates endogenous movements in total factor productivity.

To evaluate whether terms-of-trade shocks have more explanatory power for consumption volatility in a model that generates heterogeneous trade adjustment compared to the benchmark SOE-IRBC, we structure the quantitative analysis of the models into two distinct assumptions about equilibrium trade openness. First, we provide an analysis conditional on structural parameters. That is, we assume that these parameters are the same across models such that models are allowed to differ in how open the economy is in equilibrium. In this case, we find that the terms of trade are two to five times more important than in the benchmark SOE-IRBC model. Thirty-four percent of this difference is accounted for by adding monopolistic competition, sixty-two by including increasing returns to importing, and only four percent by accounting for firm heterogeneity and selection. Introducing monopolistic competition and increasing returns to importing both lower the relative price of intermediate inputs, which increases the trade openness of the economy and dominates the change in the sensitivity of the labor allocation to the final good sector to shocks. While heterogeneity and selection are crucial to match cross-sectional patterns in trade adjustment, they are inconsequential to the relative importance of the terms of trade in explaining consumption volatility.

Second, we consider an analysis in which we test to what extent the differences between the models are reduced when they are calibrated to generate the same level of equilibrium trade openness. To generate the same level of equilibrium trade openness in the different models, we allow the home bias parameter that governs the relative share of domestic to imported intermediate inputs in the production of manufacturing output to differ. Conditional on steady-state openness, we find that the quantitative predictions for the relative importance of the terms of trade of the benchmark SOE-IRBC model and the model that generates heterogeneous trade adjustment are almost identical. Hence, differences in equilibrium trade openness turn out to be the single most important factor that set the models apart. This also implies that if all the researcher is interested in is the relative importance of different shocks as drivers of aggregate consumption volatility, targeting trade openness in the benchmark SOE-IRBC framework, through the imports-to-final consumption in the data, functions as a substitute for specifying a more complex heterogeneous firms framework.

This last result is reminiscent of those in Ljungqvist & Sargent (2017) and Arkolakis et al. (2012). In the former, the elasticity of unemployment to productivity in a large class of search-and-matching models hinges on one number alone, the fundamental surplus. In the latter, the welfare change following a change in trade costs is captured in a simple formula of the change in domestic absorption and the trade elasticity in a large class of trade models. Similarly, we find that unless researchers are interested in the micro-moments of heterogeneous trade adjustment in small-open emerging economies, a simple model which is calibrated to the imports-to-consumption ratio of the economy provides a close description of the equilibrium process of aggregate variables.

This paper is related to three other strands of literature. The first studies the sources of business cycle fluctuations in emerging economies through the lens of IRBC models. TFP shocks, terms-of-trade shocks, and interest rate shocks all seem to be contributing factors to consumption volatility. For instance, Kydland & Zarazaga (2002) and Aguiar & Gopinath (2007) stress the importance of (non-)stationary TFP shocks in emerging markets, while García-Cicco et al. (2010) point that these shocks have implausible implications for the dynamics of the trade balance. Nevertheless, most papers heavily rely on TFP shocks to rationalize the observed consumption volatility. For instance, Mendoza (1995) attributes 44% to TFP shocks, also García-Cicco et al. (2010), Schmitt-Grohé & Uribe (2018), Kohn et al. (2021) and Drechsel & Tenreyro (2018) estimate that TFP shocks are responsible for respectively 95%, 86%,74% and 60% of the variation in consumption volatility. In addition, Kose (2002) and Fernández et al. (2018) attribute 12% and 25% to TFP shocks. However, all these results are obtained using the same benchmark SOE-IRBC model without heterogeneous trade adjustment. We depart from this literature by studying whether accounting for heterogeneous trade adjustment changes the importance of terms-of-trade shocks relative to TFP shocks in explaining consumption volatility.

We also contribute to the literature that studies heterogeneous trade adjustment. Kehoe & Ruhl (2008) shows how terms-of-trade shocks cannot have first-order effects on aggregate productivity in a neoclassical benchmark model. However, Amiti & Konings (2007); Goldberg et al. (2010); Gopinath & Neiman (2014); Halpern et al. (2015); Blaum et al. (2018) show that in response to terms-of-trade movements small firms change the number of imported varieties and large firms also change the imports of each previously imported product variety. To capture these patterns they introduce models of increasing returns to importing and heterogeneity which creates a connection between movements in the terms-of-trade and aggregate productivity through reallocation across heterogeneous firms. We contribute to this literature by providing a tractable general equilibrium framework that allows researchers to decompose differences between frameworks friction-by-friction and to understand whether accounting for heterogeneous adjustment matters for the relative importance of terms-of-trade shocks in explaining volatility in final consumption.

Finally, our paper ties into the literature that studies the relationship between openness and volatility of economic activity. This literature mostly focuses on explaining the relationship between the level of consumption volatility and trade openness using ad-hoc measures for trade openness such as the total trade over GDP (e.g. Koren & Tenreyro (2007), Cavallo (2009) and Giovanni & Levchenko (2009)). Like Caselli et al. (2020), we consider a theoretically grounded measure of trade openness, i.e. the imports-to-final consumption ratio, and use it to study how the importance of terms-of-trade shocks relative to TFP shocks in explaining consumption volatility changes between models with and without heterogeneous trade adjustment.

The rest of the paper is structured as follows. Section 2 develops the theoretical model we use to analyze the contribution of different shocks to consumption volatility. Section 3 provides our three main theoretical results. In section 4, we illustrate that the model developed in section 2 generates heterogeneous trade adjustment and we discuss the quantitative comparison of the different models. Finally, section 5 concludes.

2 Theoretical framework

We embed a simplified version of the Gopinath & Neiman (2014)-model in an otherwise standard SOE-IRBC model. We study an economy in which the supply side is composed of three different sectors: a downstream final good sector, an upstream manufacturing sector, and a commodities sector. We first describe how the final consumption good is produced by a final good sector that uses labor and intermediate inputs produced by the upstream manufacturing sector. Manufacturing firms produce intermediate inputs by combining labor and foreign and domestic intermediate inputs. Finally, we model the commodity sector as an endowment process. We close the economy by imposing restrictions on how consumers share risk internationally.

2.1 Technology

2.1.1 Final good sector

The final good sector consists of a representative firm that combines labor and intermediate inputs to produce the final good with a constant returns-to-scale production function:

$$Y_t = A_{St} {L_{St}}^{1-\mu} {X_{St}}^{\mu} \qquad \text{where} \qquad X_{St} = \left(\int_i {X_{it}}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

where L_{St} denotes the amount of labor used and $1-\mu$ governs the labor share in the production of services. X_{St} indicates intermediate input use and is a CES-aggregator over the individual intermediate input varieties produced by the manufacturing firms. Substitution across individual intermediate input varieties is controlled by σ. The first-order conditions that determine optimal conditional input demand are given by

$$L_{St} = (1-\mu)\frac{P_t}{W_t} Y_t, \qquad X_{St} = \mu \frac{P_t}{P_{Dt}} Y_t \qquad \text{and} \qquad X_{it} = \left(\frac{P_{it}}{P_{Dt}} \right)^{-\sigma} X_{St} \tag{2.1}$$

where P_{Dt} is the price index of domestically manufactured inputs, P_{it} is the price of domestic intermediate input variety i and W_t is the nominal wage paid to workers. The representative firm is assumed to operate in a perfectly competitive market, which together with the production function determines the final price index.[2]

$$P_t = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}} \tag{2.2}$$

2.1.2 Manufacturing sector

To produce domestic intermediate inputs, the domestic manufacturing sector combines labor and intermediate inputs as well. These intermediate inputs are either produced at home or imported. To transition from the benchmark SOE-IRBC model to the model with heterogeneous trade adjustment, we consider four different setups of the manufacturing sector: (1) the benchmark SOE-IRBC model with homogeneous firms in a perfectly competitive market; (2) a model with homogeneous firms in a monopolistically competitive market; (3) a model with homogeneous firms in a monopolistically competitive market producing under increasing returns to importing; (4) the model with heterogeneous firms which generates heterogeneous trade adjustment. Here we discuss the model that generates heterogeneous trade adjustment and leave the details for the other models in the Appendix.

[2] Modelling the relation between the final good and the manufacturing sector as vertical provides a parsimonious way to match the pattern that final consumer prices are much less responsive to nominal exchange rate movements compared to import prices or producer prices (e.g. Burstein & Gopinath (2014)). In this way, the final good sector might be viewed as a distribution sector that combines final manufacturing products with local labor inputs to deliver the final good to consumers

Production technology There is a continuous unit measure of domestic manufacturing firms indexed by i. Domestic firms produce using the following Cobb-Douglas production function:

$$Y_{it} = A_{Dt}\varphi_i {L_{Dit}}^{1-\gamma} {X_{Dit}}^{\gamma}$$

where firm i's productivity level is a combination of its time-invariant productivity φ_i and A_{Dt} which is a sector-level TFP shock process in the manufacturing sector. L_{Dit} and X_{Dit} represent productive labor use and intermediate input use respectively and γ is the intermediate input share in production. The intermediate input bundle is a CES aggregate of foreign and domestic intermediate input bundles:

$$X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} {Q_{Dit}}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} {Q_{Mit}}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where Q_{Dit} and Q_{Mit} represent firm i's use of domestic and imported intermediate inputs respectively and ε determines the degree of substitutability between the domestic and foreign input bundles. ω is a home-bias parameter that determines the extent to which manufacturing firms prefer domestic intermediate input conditional on relative intermediate input prices. Finally, domestic and imported input bundles are CES aggregates of individual domestic and foreign intermediate input varieties.

$$Q_{Dit} = \left(\int_j {q_{Dijt}}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad Q_{Mit} = \left(\int_{k\in|\mathscr{L}_{it}|} {q_{Mikt}}^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}} \tag{2.3}$$

The domestic intermediate input bundle aggregates the varieties produced by the domestic manufacturing sector. The quantity used of the output of firm j by firm i is denoted by q_{Dijt} and substitution among these varieties depends on σ. Substitution across imported input varieties is governed by the elasticity θ. Following Gopinath & Neiman (2014), we assume that individual imported varieties are indistinguishable from one another in their quality or source. Under this assumption, there is a common dollar price $P^{\$}_{Mt}$ for all imported varieties k and the firm-specific imported intermediate input bundle price is the following

$$P_{Mit} = E_t P^{\$}_{Mt} |\mathscr{L}_{it}|^{\frac{1}{1-\theta}}$$

where E_t is the nominal exchange rate at time t. The firm-specific intermediate input price is

$$P_{Xit} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} |\mathscr{L}_{it}|^{\frac{1-\varepsilon}{1-\theta}}\right)^{\frac{1}{1-\varepsilon}}$$

where $P_{Mt} = E_t P^{\$}_{Mt}$. In the setups without increasing returns to importing it follows that $|\mathscr{L}_{it}| = 1\ \forall i, t$ while in setups with increasing returns to importing the measure of imported varieties is optimally chosen. Also, it is allowed to be zero for firms that optimally choose not to import.

Market structure The manufacturing sector sells both to itself and to the services sector. Because manufacturing firms substitute across domestic intermediate inputs with the same elasticity of substitution as the services sector[3], final demand for manufacturing output is given by:

$$Y_{it} = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})$$

where $Q_{Dt} \equiv \int_i Q_{Dit} di$ is the total demand for manufacturing output from domestic manufacturers. The domestic manufacturing price index is a CES aggregate of domestic variety prices $P_{Dt} = \left(\int_i {P_{it}}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$. We assume that manufacturers compete under monopolistic competition which combined with CES demand for manufacturing output leads to a pricing rule that consists of a constant markup over marginal costs.[4]

$$P_{it} = \frac{\sigma}{\sigma - 1}\text{MC}_{it}$$

Optimal input allocation Conditional on choosing the measure of imported varieties $|\mathscr{L}_{it}|$, we derive the firm's marginal cost function by solving the firm's cost minimization problem. The first-order conditions for conditional input demand are the following.

$$L_{Dit} = (1-\gamma)\frac{\text{MC}_{it}}{W_t} Y_{it} \quad \text{and} \quad X_{Dit} = \gamma \frac{\text{MC}_{it}}{P_{Xit}} Y_{it} \tag{2.4}$$

Optimal demand for domestic and imported bundles is governed by the first-order conditions of the second-tier problem of manufacturing producers and depends on the elasticity of substitution between input bundles.

$$Q_{Dit} = \omega \left(\frac{P_{Dt}}{P_{Xit}}\right)^{-\varepsilon} X_{Dit} \quad \text{and} \quad Q_{Mit} = (1-\omega)\left(\frac{P_{Mit}}{P_{Xit}}\right)^{-\varepsilon} X_{Dit}$$

Finally, the optimal demand for each type of variety is pinned down by the first-order conditions of the third tier of the manufacturing producer's problem.

$$q_{Dijt} = \left(\frac{P_{jt}}{P_{Dt}}\right)^{-\sigma} Q_{Dit} \quad \text{and} \quad q_{Mikt} = \left(\frac{P_{Mkt}}{P_{Mit}}\right)^{-\theta} Q_{Mit}$$

[3]This follows a large literature in closed and open economy macroeconomics (Nakamura & Steinsson (2010), Gopinath & Neiman (2014) and Blaum et al. (2018)).

[4]By assuming that the manufacturing sector charges a constant markup over marginal costs, we deviate from recent literature in international macroeconomics that accounts for pricing-to-market by allowing for more general forms of competition (e.g. Amiti et al. (2019) and Gopinath et al. (2020)). However, in Appendix **??** we show that, in contrast to developed economies where the terms-of-trade is less volatile than the real exchange (Atkeson & Burstein (2008)), commodity exporters experience the opposite. Because assuming monopolistic competition does not compromise the model in fitting this empirical fact, we abstract from pricing to market. In the setup under perfect competition, we evaluate the model in the limit where $\sigma/(\sigma-1) \to 1$ and manufacturing prices are equal to marginal costs.

Combining these expressions with the production function, manufacturing firms' marginal cost function conditional on a sourcing strategy $|\mathscr{L}_{it}|$ is the following.

$$\mathrm{MC}_{it}\left(|\mathscr{L}_{it}|\right) = \frac{1}{A_{Dt}} \frac{1}{\varphi_i} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}$$

Optimal sourcing decision Without increasing returns to importing, the optimal sourcing strategy is $|\mathscr{L}_{it}| = 1$. Under increasing returns to importing, firms weigh the benefits of an additional imported intermediate input variety with the additional fixed costs necessary to source it. This fixed cost is paid every period in domestic labor units, such that total fixed costs are $W_t f |\mathscr{L}_{it}|$ where f is the labor requirement per imported variety. Manufacturing firms maximize profits $(P_{it} - \mathrm{MC}_{it}(|\mathscr{L}_{it}|))\, Y_{it}$ net of fixed costs. To obtain an explicit solution for the measure of imported varieties, we assume that $\varepsilon = \theta$ such that the fixed costs to be paid are linear in the measure.[5] Under these restrictions, the optimal number of intermediate input varieties is

$$|\mathscr{L}_{it}| = \frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]$$

where φ_{Mt} is the cutoff productivity level defined by equating revenues to fixed costs, such that $|\mathscr{L}_{it}(\varphi_{Mt})| = 0$.[6] Plugging in the cutoff definition and the optimal number of imported intermediate input varieties, we re-express input prices solely as a function of aggregate variables and the firm-level productivity level:

$$P_{Xit} = {\gamma_{Dit}}^{\frac{1}{\varepsilon-1}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt} \quad \text{where} \quad \gamma_{Dit} = \begin{cases} \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} & \text{if } \varphi_i \geq \varphi_{Mt} \\ 1 & \text{otherwise} \end{cases}$$

where γ_{Dit} is the domestic intermediate input share which is decreasing in φ_i if $\gamma(\sigma-1) < \varepsilon - 1$ such that the measure is increasing in firm-level productivity.

2.1.3 Commodity sector

We follow Fernández et al. (2018) and model the commodity sector as an endowment process that is the only source of foreign income for the economy. We make this simplifying assumption for two reasons. First, it is plausible that world commodity prices are exogenously given to the respective economies we consider. For instance, take Colombia and Chile as two representative countries.

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[5] This assumption is also imposed in Gopinath & Neiman (2014) and in section 4 we show that these simplifications do not compromise the model's ability to match the key empirical patterns.

[6] The expression for φ_{Mt} is the following

$$\varphi_{Mt} = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left[\frac{\gamma}{\varepsilon-1}(1-\omega)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}\right]^{-\frac{1}{\sigma-1}} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}{P_{Mt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \left[\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

While oil represents roughly 60% of Colombia's total exports, Colombia was only the 20^{th} largest oil producer in 2020, according to the US Energy Information Administration. Also, Colombia has never been a member of OPEC. Chile accounted for a little under 10% of the world's raw copper production in 2015 according to the US Geological Survey 2017 but copper represents more than half of its exports. Second, adjusting physical commodity output is often hard to achieve in the short run due to significant time-to-build in extraction capacity.[7] For these reasons, income from commodity exports is arguably well approximated by an endowment process that keeps physical output fixed in the short run but accounts for income fluctuations stemming from changes in world commodity prices. These restrictions imply that we discard the reallocation of labor in and out of the commodity sector at business cycle frequency.

2.2 Final demand

The economy is populated by a representative consumer that buys services and supplies labor inelastically.[8] For simplicity, we assume that consumers cannot share risks internationally and that the economy is in financial autarky. In financial autarky, consumers consume their full income and the real exchange rate adjusts to ensure that the value of commodity exports equals the value of imported intermediate inputs when expressed in terms of the domestic good such that trade is balanced each period. Formally, we have that:

$$TB_t = E_t P^{\$}_{Xt} X + W_t L + \Pi_t - P_t C_t = 0$$

where Π_t are profits paid out to consumers by firms in the manufacturing sector and $P_t C_t$ is the total expenditure on services in any given period t.

2.3 Equilibrium

Definition 1 (Stable equilibrium)**.** *Given the set of deep parameters* $\Theta = \left\{\gamma, \omega, \varepsilon, \sigma, \theta, \kappa, \underline{\Phi}, \delta_1, \delta_2, R^{\$}, P^{\$}_M, P^{\$}_X, X, f\right\}_{t=0}^{\infty}$ *and a set of exogenous processes* $\left\{P^{\$}_{Xt}, P^{\$}_{Mt}, A_{Dt}, A_{St}\right\}_{t=0}^{\infty}$*, a stable equilibrium is a set of price processes* $\{P_{Dt}, W_t, E_t\}_{t=0}^{\infty}$ *that ensures that the equilibrium processes for the endogenous variables* $\left\{C_t, Y_t, X_{St}, Q_{Dt}, L_{St}, L_{Dt}, L_{Mt}, Q_{Mt}\right\}_{t=0}^{\infty}$ *satisfy the following conditions (1) Consumers maximize utility given the budget constraint, (2) final good and manufacturing producers maximize profits and (3) markets clear:*

[7] For instance, Asker et al. (2019) model oil extraction through a Leontief production function in labor and extractive capital that is pre-determined in the short run. Hence, without additional investment in physical extraction capacity, there is no reallocation of productive labor to the commodities sector.

[8] Doing so, we implicitly assume that domestic financial markets provide full insurance against idiosyncratic shocks across households.

Goods market clearing

$$Y_t = C_t, \qquad Y_{Dit} = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})$$

Labor market clearing

$$L = L_{St} + \int_i (L_{Dit} + L_{Mit})\, di$$

Current account

$$TB_t = W_t L + \Pi_t + E_t P^{\$}_{Xt} X - P_t C_t = 0$$

Finally, we normalize the price of the final good sector: $P_t = 1$.

Defining trade openness In all models we consider, the equilibrium conditions can be written in terms of an auxiliary variable H_t, which we call the "trade openness" of the economy. It is defined by rewriting the expression for imports $W_t L + \Pi_t - P_t C_t$ as proportional to final consumer spending $P_t C_t$ (see Appendix B). The trade balance equation is then rewritten as follows.

$$TB_t = E_t P^{\$}_{Xt} X + \underbrace{W_t L + \Pi_t - P_t C_t}_{\text{Imports}} = E_t P^{\$}_{Xt} X - \underbrace{\mu\gamma \frac{\sigma - 1}{\sigma} H^m_t (\Theta)\, P_t C_t}_{\text{Imports}}$$

As indicated by the superscript m, the exact expression of H_t differs across the models we consider. However, trade openness is always bounded between zero and one and captures the degree to which the economy depends on imported intermediate inputs to produce final consumption.[9]

Existence and uniqueness of the equilibrium One reason why rewriting the equilibria in terms of H_t is useful is because the non-linear equilibria of all the models are implicitly defined as a fixed point in trade openness. Moreover, the following proposition shows that, apart from the model with heterogeneous trade adjustment, the equilibria are certain to exist and to be unique.

Proposition 1 (Existence and uniqueness of the equilibra)**.** *For each of the models, the equilibrium*

[9] For instance, in the model with homogenous manufacturers that compete under monopolistic competition without increasing returns, H^{MC}_t is defined below. Also, the share spent on imported intermediate inputs relative to all intermediate input spending is increasing in trade openness:

$$F^{\text{MC}}_t = \frac{1}{1 + \left(1 - \gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\left(\frac{E_t P^{\$}_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}, \qquad S^M_t \equiv \frac{P_{Mt} Q_{Mt}}{P_{Xt} X_{Dt}} = \frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right) H_t}{1-\gamma\frac{\sigma-1}{\sigma} H_t}$$

H_t intuitively depends on the relative input price of foreign and domestic intermediate inputs and the home-bias parameter ω. For small values of ω manufacturing producers are more dependent on imported inputs and H_t is closer to one. The same is true when the price of domestic inputs in domestic currency is high relative to that of imported intermediate inputs.

can be solved via a model-specific fixed point equation in trade openness H_t^m*, given by:*

$$F^m\left(H_t^m;\Theta\right)=0 \qquad \forall\ m\in\{IRBC, MC, IRS, HTA\}$$

In addition, the equilibria defined H_t^m *by* $F^m(\cdot)$ *exist and are unique* $m \in \{IRBC, MC, IRS\}$

Proof. See Appendix C. □

The argument behind this result is that for each model m we can construct a function $F^m\left(H_t^m;\Theta\right)$ such that $\lim_{H_t^m\to 0} F^m\left(H_t^m;\Theta\right) = -1$ and $\lim_{H\to 1} F^m(H;\Theta) = \infty$. By Bolzano's Theorem, there exists at least one root $H_t^m \in (0,1)$. The uniqueness of the steady state follows from the fact that $F^m(H_t^m,\Theta)$ is monotonically increasing in $H_t^m \in (0,1)$.[10] The same argument cannot be used in the heterogeneous model with arbitrary levels of heterogeneity. Nevertheless, when heterogeneity approaches the upper limit, $\kappa \to \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}$, the limit where model's moments remain finite, the same argument can be used and the steady state exists and is unique. When the model converges to an economy with a degenerate productivity distribution, $\kappa\to\infty$, the model collapses to the homogenous firm model with increasing returns to scale on the importing bundle for which proposition 1 ensures existence and uniqueness. Therefore, we conjecture that the steady state also exists and is unique in the intermediate heterogeneity cases. To support this claim, Figure 1 shows for different values of trade openness the value of the non-linear function that makes up the fixed point equation in each of the models. This figure illustrates that the implicit functions of the homogeneous and heterogeneous firm versions of models with increasing returns to importing behave very similarly.

Figure 1 highlights how the different models deliver different equilibrium levels of trade openness. First, when the manufacturing sector operates under monopolistic competition, domestic intermediate input prices are higher than in the SOE-IRBC benchmark, where they are priced at the marginal cost of production. This incentivizes domestic manufacturers to substitute domestic intermediate inputs for intermediate inputs sourced from abroad. As a result, there is a departure from the efficient allocation, as the manufacturing sector produces less than under the perfectly competitive benchmark, and the equilibrium trade openness of the economy rises.

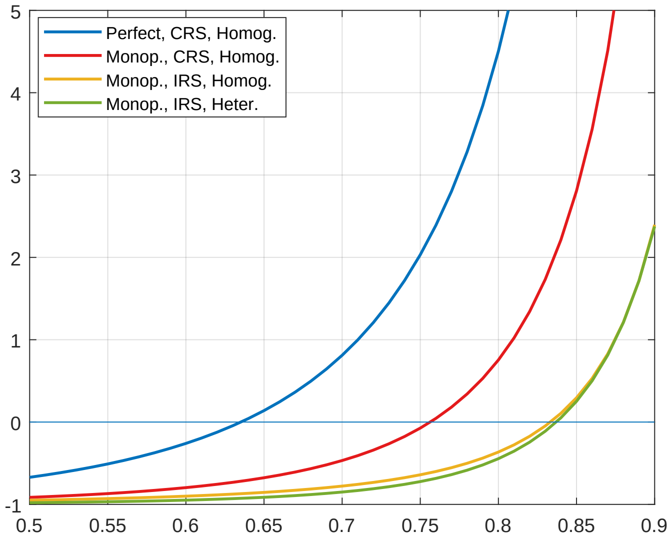
Second, the model with increasing returns to importing delivers an economy with higher trade openness compared to an economy without increasing returns when the fixed cost of sourcing additional product varieties is not too large. The introduction of increasing returns to importing changes the sourcing problem in two ways. On the one hand, the love-for-variety aggregator provides incentives to lower marginal costs by increasing the set of imported intermediate input

[10]For example, in the case of monopolistic competition, it follows that

$$F^{\mathrm{MC}}(H,\Theta)=\Lambda^{\mathrm{MC}}(\Theta)\,\frac{H^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_2\left[\chi_2-\mu\gamma H\right](1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}}-1$$

where $\Lambda^{\mathrm{MC}}(\Theta)$ is a function of the structural parameters.

Figure 1: Equilibrium fixed point equation $F^m(H;\Theta)$ for different models



Notes: This figure plots the fixed point equation $F^m(H(\Theta);\Theta)$ that determines the equilibrium trade openness in each of the models for the different models separately. This function is evaluated at the baseline calibration discussed in section 4.

varieties. On the other hand, using more intermediate input varieties requires higher fixed costs as each variety carries a per-variety fixed cost. When the per-variety fixed cost approaches zero, the benefits of adding intermediate input varieties increasingly outweigh the costs of accessing them, leading to higher equilibrium trade openness.[11]

Finally, trade openness further rises with the introduction of firm-level heterogeneity. As long as $\varepsilon - 1 > \gamma(1-\sigma)$, larger firms will source more intermediate input varieties. As manufacturing firms produce with a production technology that is characterized by love-for-variety on the imported intermediate input bundle, larger firms can reduce their marginal cost more and attract a larger market share. This positive correlation between importing and market share leads to a more open economy in the aggregate, albeit only to a limited extent.

3 Theoretical results

In this section, we consider the model's first-order dynamic solutions around the model's steady state. In particular, we assume that the exogenous stochastic processes $\{a_t, a_{Dt}, p^{\$}_{Xt}, p^{\$}_{Mt}\}$ follow shock-specific AR(1)-processes, which are not model-specific. We provide three key results. First, the different models give rise to the same goods and labor market clearing conditions that relate final consumption and the real exchange rate to exogenous shocks and changes in trade openness. The differences between the models are fully captured by the elasticities that pre-multiply changes in trade openness in each of the equations. Second, the contribution of terms-of-trade shocks

[11]Because calibrated values in Gopinath & Neiman (2014) and Halpern et al. (2015) are very small and not very far from zero, we consider this limiting result as the relevant limit.

relative to productivity shocks in explaining consumption volatility is pinned down by one general equilibrium elasticity, which we will refer to as the terms-of-trade elasticity. Third, the terms-of-trade elasticity can always be written as the imports-to-consumption ratio times a distortion term.

3.1 General structure of goods and labor markets

Theorem 1 shows that across all the models the equilibrium in the goods and labor market can be represented by two equations that relate changes in final consumption and in the real exchange rate to trade openness. The models deliver the same equilibrium relationship between the endogenous variables and the shocks and only differ in terms of the two partial equilibrium elasticities that govern the direct relationship between endogenous openness, changes in final consumption, and the real exchange rate.

Theorem 1 (General Structure)**.** *Across all models, the equilibrium in labor and goods markets reduces to two equations that express how changes in final consumption and the real exchange rate relate to changes in openness and exogenous shocks. They are given by:*

$$c_{St} = \frac{\mu}{1-\gamma} a_{Dt} + a_{St} + \nu^m_{cH}\left(H;\tilde{\Theta}\right)\eta_t \tag{3.1}$$

$$\eta_t = \frac{1}{\nu^m_{qH}\left(H;\tilde{\Theta}\right)}\left(\frac{1-\mu}{1-\gamma} a_{Dt} - a_{St} + p^{\$}_{Mt} + q_t\right) \tag{3.2}$$

where $q_t \equiv e_t - p_{St}$ is the real exchange rate and η_t is the deviation from steady state trade openness in percentage changes. Moreover, we have that $\nu^m_{cH}\left(H;\tilde{\Theta}\right) > 0$ and $\nu^m_{qH}\left(H;\tilde{\Theta}\right) < 0$.

Proof. See Appendix D. □

The first equation captures how changes in productivity of domestic factors and foreign factors translate into changes in final consumption. To obtain this equation, we combine the linearized expressions for product market clearing and labor market clearing equation, which together yield:

$$c_{St} = w_t - p_{St} + \nu^m_{lH}\left(H;\tilde{\Theta}\right)\eta_t$$

This expression implies that changes in final consumption are determined by changes in real wages and changes in trade openness. On the one hand, changes in real wages reflect changes in the productivity of labor as a domestic factor to produce final goods. On the other hand, changes in trade openness represent changes in the reliance on foreign intermediate inputs which induces reallocation of labor towards the final good sector. The sensitivity of the downstream labor allocation is captured by $\nu^m_{lH}\left(H;\tilde{\Theta}\right)$.[12] Real wages can be expressed as a function of only shocks

[12] In Appendix D we show that $\nu^m_{lH}\left(H;\tilde{\Theta}\right)$ governs how the labor allocation to the final good sector responds to changes in trade openness.

and trade openness as well by combining the linearized price level of the final good sector and the linearized price level of the manufacturing sector:

$$w_t - p_{St} = \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right), \qquad p_{Dt} - p_{St} = a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu^m_{pH}\left(H;\tilde{\Theta}\right)\eta_t$$

In response to an increase in total factor productivity in the production of the final good, real wages rise to reflect the higher marginal product of labor in producing the final good. They fall with an increase in the real price of manufacturing goods, indicating the lower marginal product of labor due to substitution from intermediate inputs towards labor. Real manufacturing prices decrease with a rise in total factor productivity in the manufacturing sector and increase with positive shocks to total factor productivity in producing the final good, through the equilibrium response of real wages. Following an increase in trade openness, real manufacturing prices drop, reflecting the increased use of imported intermediate input relative to domestically produced. The extent to which real manufacturing prices respond to changes in trade openness is captured by ν_{pH}, which differs across the models. Combining these last two expressions with the linearized labor market clearing expression delivers the first equation in Theorem 1.

The second equation describes how trade openness changes in response to shocks and changes in the real exchange rate, capturing expenditure switching between domestic and foreign intermediate inputs.[13] To arrive at this equation, we combine the linearized model-specific definition of trade openness and the expression for the productivity cut-off and obtain, for instance, for the model with increasing returns to importing[14]:

$$-\underbrace{(\varepsilon-1)\left(1-H^{\mathrm{IRS}}\right)\left(p^{\$}_{Mt} + q_t - \left(p_{Dt} - p_{St}\right)\right)}_{\text{Substitution channel}} =$$
$$\underbrace{\left(\underbrace{\frac{1-\gamma}{\gamma}\frac{\varepsilon-1}{1-\mu}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H^{\mathrm{IRS}}}{1-\gamma^{\frac{\sigma-1}{\sigma}}H^{\mathrm{IRS}}}\right)\nu^{\mathrm{IRS}}_{pH}\left(H;\tilde{\Theta}\right)}_{\text{Profits}} - \underbrace{\nu^{\mathrm{IRS}}_{lH}\left(H;\tilde{\Theta}\right)}_{\text{Fixed costs}}\right)\left(1-H^{\mathrm{IRS}}\right)\eta_t}_{\text{IRS channel}}$$

This equation captures two channels that determine the degree of expenditure switching in the model. First, in all models, there is a substitution channel that arises because of cost-minimization by manufacturing firms. When manufacturers choose the bundle of intermediate input that delivers

[13] In contrast to the literature in which final demand is an aggregator over domestic and imported final consumption goods (e.g. Obstfeld & Rogoff (1995),Galì & Monacelli (2005) and Itskhoki & Mukhin (2021)), our expenditure switching channel stems from optimal input allocation and substitution across intermediate inputs on the supply side as in Obstfeld (2001).

[14] Given that the expressions for H^m_t are model specific, we illustrate the steps with the homogeneous firms model as it captures the substitution and increasing returns to importing channel of expenditure switching well. The heterogeneous firms model has a similar, albeit more convoluted, expression.

the lowest marginal costs for a given quantity of output, their decision depends on the relative price of foreign inputs to domestic inputs, $p^{\$}_{Mt} + q_t - (p_{Dt} - p_{St})$, the elasticity of substitution between domestic and imported intermediate inputs and the pool of the domestic intermediate input suppliers, captured through H. Second, in the models with increasing returns to importing, manufacturing firms also solve a profit maximization problem. Doing so, firms decide on how many intermediate input varieties to source from abroad by weighing the additional profits, through lowering marginal costs, with the additional fixed costs associated with importing more varieties. In response to shocks, the pass-through from changes in real manufacturing prices into aggregate manufacturing profits is captured by the coefficient on $\nu^m_{pH}(H;\tilde{\Theta})$ and the degree by which demand for manufacturing output changes relative to how per variety fixed costs changes, is captured by $\nu^m_{lH}(H;\tilde{\Theta})$. The heterogeneous firm model admits the same structure, but the difference lies in the coefficient on $\nu^m_{pH}(H;\tilde{\Theta})$ that now also the fact that not all firms in the economy will access the IRS technology, which changes how changes in real manufacturing prices pass into profits. After plugging in the expression for changes in real manufacturing prices, we arrive at equation 3.2.

3.2 Relative importance of terms-of-trade shocks

We use this common structure across models to derive the equilibrium processes for consumption and the real exchange rate and the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining the variance of final consumption. In the absence of international risk-sharing possibilities, trade must be balanced in each period. Combining the linearized trade balance equation with the general structure of goods and labor markets leads to the following equilibrium processes for changes in consumption and in the real exchange rate:

Corollary 1 (Equilibrium processes - Financial Autarky)**.** *In financial autarky, the equilibrium processes of final consumption and the real exchange rate as a function of the exogenous shocks are given by:*

$$\begin{aligned} c_{St} &= a_{St} + \frac{1}{1-\gamma}\left(\mu - \nu^m_c(H;\tilde{\Theta})\right) a_{Dt} + \nu^m_c(H;\tilde{\Theta})\left(p^{\$}_{Xt} - p^{\$}_{Mt}\right) \\ q_t &= a_{St} - \frac{1}{1-\gamma}\left((1-\mu) - \nu^m_q(H;\tilde{\Theta})\right) a_{Dt} - \nu^m_q(H;\tilde{\Theta})\, p^{\$}_{Xt} - \left(1 - \nu^m_q(H;\tilde{\Theta})\right) p^{\$}_{Mt} \end{aligned}$$

where

$$\nu^m_c(H;\tilde{\Theta}) \equiv \frac{\nu^m_{cH}(H;\tilde{\Theta})}{1 + \nu^m_{cH}(H;\tilde{\Theta}) - \nu^m_{qH}(H;\tilde{\Theta})}, \quad \nu^m_q(H;\tilde{\Theta}) \equiv -\frac{\nu^m_{qH}(H;\tilde{\Theta})}{1 + \nu^m_{cH}(H;\tilde{\Theta}) - \nu^m_{qH}(H;\tilde{\Theta})}$$

and where $\nu^m_c(H;\tilde{\Theta}) > 0$ *and* $\nu^m_q(H;\tilde{\Theta}) > 0$ *following Theorem 1.*

Proof. See Appendix E. □

Following Corollary 1, any differences between frameworks can be thought of as differences in

$\nu_c^m\left(H;\tilde{\Theta}\right)$, which we will refer to as the terms-of-trade elasticity as it determines how final consumption responds to terms-of-trade shocks, and $\nu_q^m\left(H;\tilde{\Theta}\right)$. Also, from Corollary 1, it is immediate that models that have a higher terms-of-trade elasticity will put more weight on terms-of-trade shocks as a source for consumption movements and less weight on exogenous manufacturing TFP shocks. Importantly, the extent to which different models will have different predictions for the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining the variance of final consumption is solely determined by the terms-of-trade elasticity $\nu_c^m\left(H;\tilde{\Theta}\right)$.

Theorem 2 (Terms-of-trade relative to TFP)**.** *Under financial autarky, the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining the variance of final consumption is given by:*

$$\frac{\mathbb{V}\left(c_{St}|p^{\$}_{Mt},p^{\$}_{Xt}\right)}{\mathbb{V}\left(c_{St}|a_{Dt},p_{St}\right)}=\frac{\sigma_X^2}{\sigma_A^2}\frac{\left(\nu_c^m\left(H;\tilde{\Theta}\right)\right)^2}{\frac{\sigma_S^2}{\sigma_D^2}+\left(\frac{\mu-\nu_c^m\left(H;\tilde{\Theta}\right)}{1-\gamma}\right)^2}$$

where σ_i^2's are the variances of the shock processes. In addition, the relative importance of terms-of-trade shocks is rising in $\nu_c^m\left(H;\tilde{\Theta}\right)$, that is

$$\partial\frac{\mathbb{V}\left(c_{St}|p^{\$}_{Mt},p^{\$}_{Xt}\right)}{\mathbb{V}\left(c_{St}|a_{Dt},p_{St}\right)}\Big/\partial\nu_c\left(H;\tilde{\Theta}\right)>0$$

Proof. Follows directly from applying the unconditional, $\mathbb{V}(\cdot)$, and conditional, $\mathbb{V}(\cdot|)$, variance operators to the expression for c_{St} in Corollary 1. □

While theorem 1 provides a unifying framework for the SOE-IRBC benchmark and the model with heterogeneous trade adjustment, Theorem 2 illustrates that, to understand whether different models have different predictions for the relative importance of terms-of-trade shocks in explaining consumption volatility, all we need to know is the extent to which models have different predictions regarding the terms-of-trade elasticity.

3.3 The terms-of-trade elasticity

It turns out to be difficult to rank the models in terms of their predictions for the terms-of-trade elasticity ex-ante. Nonetheless, we now provide intuition into how the terms-of-trade elasticity differs across the models. In particular, the following proposition establishes that we can always write the terms-of-trade elasticity as a combination of two distinct elements.

Proposition 2 (Decomposing the terms-of-trade elasticity)**.** *The terms-of-trade elasticity $\nu_c^m\left(H;\tilde{\Theta}\right)$ has the following common structure across frameworks.*

$$\nu_c^m\left(H^m(\Theta);\tilde{\Theta}\right)=\underbrace{\mu\gamma H^m(\Theta)}_{\textit{Imports-to-consumption}}\cdot\underbrace{\Xi^m\left(H^m(\Theta),\tilde{\Theta}\right)}_{\textit{Distortion}}$$

where $\Xi^m(\cdot) = 1$ *in the benchmark SOE-IRBC model.*

Proof. See Appendix E. □

We refer to the first part of $\nu_c^m\left(H^m(\Theta);\tilde{\Theta}\right)$ as the "Imports-to-consumption" term and to the second part as the "distortion" term. The imports-to-consumption term is simply the product of the intermediate input elasticity in the final good sector μ, the intermediate input elasticity in the manufacturing sector γ, and the steady-state equilibrium trade openness level H. When H approaches zero, the economy is closed and relies minimally on imports for production. In this case, the portion of consumption variance explained by terms of trade movements approaches zero because import price shocks do not affect input decisions and the exchange rate insulates the economy completely from volatility in export prices by adjusting in the opposite direction. Conversely, as the economy relies more on imported intermediate inputs and opens up, such that H approaches 1, the share in consumption volatility explained by terms-of-trade shocks rises. We allude to the second term as the distortion term because the distortion term is equal to one in the benchmark SOE-IRBC model. However, once the benchmark SOE-IRBC model is enriched with frictions to capture heterogeneous trade adjustment, the importance of terms-of-trade shocks will also depend on the distortion term.

To understand how the two terms arise, we start by unpacking the building blocks of its numerator $\nu_{cH}\left(H^m(\Theta);\tilde{\Theta}\right)$, which is given by[15]:

$$\nu_{cH}\left(H^m(\Theta);\tilde{\Theta}\right) = \frac{\mu}{1-\mu}\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right) + \nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$$

where $\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right)$ captures how manufacturing prices move with openness and where $\nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$ captures how the labor allocation to the final good sector moves with openness.[16] In particular, $\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right)$ is given by:

$$\frac{\mu}{1-\mu}\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right) \equiv \underbrace{\underbrace{\mu\gamma H^m}_{\text{Imports-to-consumption}} \cdot \underbrace{\frac{1}{1-\gamma}}_{\substack{\text{roundabout}\\ \text{production}}} \cdot \underbrace{\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H^m}{1-\gamma^{\frac{\sigma-1}{\sigma}}H^m}}_{\text{Import share}}}_{\text{Exposure}}$$
$$\cdot \underbrace{\underbrace{\frac{1}{H^m(1-H^m)(\varepsilon-1)}}_{\text{No selection}} \underbrace{\frac{1}{\gamma(\sigma-1)}\frac{\tilde{\kappa}H(1-H^m)(\varepsilon-1)}{H^m(1-\tilde{\kappa}H)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)+H^m(1-H^m)\gamma^{\frac{\sigma-1}{\sigma}}}}_{\text{Selection}}}_{\text{Substitution}}$$

The components of $\frac{\mu}{1-\mu}\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right)$ can be separated into two main components. First, the "exposure" term is common across models. Apart from the imports-to-consumption term, the extent to which real manufacturing prices are exposed to changes in openness depends on two

[15] See Appendix (D) for more detail on the derivations.

[16] Both were defined in section 3.1

additional terms. On the one hand, the presence of roundabout production makes exposure of manufacturing prices to changes in trade openness depend on the intermediate input elasticity in manufacturing γ. On the other hand, exposure additionally depends on the steady-state share of imported intermediate input to total intermediate input spending in manufacturing. Both the extent to which final demand changes and the imported intermediate input share rise in trade openness.

Second, the "substitution term" differs across models with and without an active firm-extensive margin. In the absence of a firm-extensive margin, the substitution term depends on H^m and the elasticity of substitution between intermediate inputs ε. When the latter is high, domestic inputs are good substitutes for imported inputs and so manufacturing firms can easily substitute if import prices are high, insulating p_{Dt} from foreign shocks. This micro elasticity $\varepsilon - 1$ is adjusted by $1/(H^m(1-H^m))$ to form a macro elasticity of substitution, where the latter weighs the relative supply of domestic and imported inputs in equilibrium. The higher H^m, the smaller the pool of domestically produced intermediate inputs and the lower aggregate substitution becomes. In the model with heterogeneity and an active extensive margin, the substitution term is modified by the "selection" term. As κ goes towards its lower limit and productivity draws become more heterogeneous, the market share allocated to highly productive firms grows.[17] Because very large firms also adjust their imports more on the firm-sub-intensive margin and less on the firm-sub-extensive margin, the relevant macro elasticity changes. This is reflected in the fact that as κ goes towards its lower limit and $\tilde{\kappa}$ goes to one[18], the micro-elasticity of the no-selection part, $\varepsilon - 1$, is replaced with the lower $\gamma(\sigma - 1)$.[19]. In line with Gopinath & Neiman (2014), the macro elasticity in the model with heterogeneous select firms and selection is always higher than in the model without heterogeneity and selection.

The expression for $\nu_{cH}\left(H^m(\Theta);\tilde{\Theta}\right)$ also depends on $\nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$. In Appendix B, we show that the change in the labor allocation to the final good sector can be written solely as a function of changes in trade openness such that:

$$l_{St} = \underbrace{\underbrace{\mu\gamma H}_{\text{Imports-to-consumption}} \cdot \underbrace{\frac{1}{\chi^m(\tilde{\Theta}) - \mu\gamma H^m}}_{\propto \text{ employment share}}}_{\equiv \nu_{lH}(H^m(\Theta);\tilde{\Theta})} \eta_t$$

where $\chi^m(\tilde{\Theta})$ is a combination of deep parameters which is different across the models. Because of the Cobb-Douglas structure, $\nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$ is solely composed of an exposure term and has two parts. First, like before, the sensitivity of the labor allocation to the final good sector in response to changes in trade openness depends on the imports-to-consumption ratio. Second, the sensitivity

[17] Recall that $\varepsilon - 1 > \gamma(\sigma - 1)$ is necessary for the model to produce finite moments.

[18] $\tilde{\kappa}$ is a combination of the subset of deep parameters $\tilde{\Theta}$.

[19] See Chaney (2008) for a similar argument about how the relevant micro elasticity of substitution changes depending on how the importance of the firm-intensive and firm-extensive margin in changes in trade flows

of the labor allocation to the final good sector also depends on a term that is proportional to the share of the labor allocation to the final good sector in the steady state. On the one hand, the steady-state labor allocation to the final good sector rises with trade openness, capturing the fact that as manufacturing firms increasingly rely on imported intermediate inputs, they substitute away from labor which flows to the final good sector. On the other hand, the steady-state labor allocation to the final good sector also depends on $\chi^m(\tilde{\Theta})$. While $\chi^m(\tilde{\Theta}) = 1$ in the benchmark SOE-IRBC model, $\chi^m(\tilde{\Theta})$ changes discontinuously between different models, making it hard to determine ex-ante whether distortions that work through $\nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$ will amplify or dampen the importance of terms-of-trade shocks relative to the benchmark SOE-IRBC model.[20]

The analysis of $\nu_{cH}\left(H^m(\Theta);\tilde{\Theta}\right)$ highlights that because the imports-to-consumption ratio is present in both components of $\nu_{cH}\left(H^m(\Theta);\tilde{\Theta}\right)$, it is also one part of the general equilibrium $\nu_c^m\left(H^m(\Theta);\tilde{\Theta}\right)$. The remaining elements of $\nu_{pH}\left(H^m(\Theta);\tilde{\Theta}\right)$ and $\nu_{lH}\left(H^m(\Theta);\tilde{\Theta}\right)$, such as the substitution term and the term proportional to the steady-state labor allocation to the final good sector, collectively make up the distortion term after being adjusted by $(1 + \nu_{cH}^m\left(H^m(\Theta);\tilde{\Theta}\right) - \nu_{qH}^m\left(H^m(\Theta);\tilde{\Theta}\right))^{-1}$ to account for how openness itself moves with exogenous shocks in general equilibrium. While the imports-to-consumption ratio straightforwardly depends on steady-state trade openness across the models, the distortion term depends in a more complicated way on the specifics of each of the models. This precludes us from making ex-ante predictions for the different models and to understand to a full extent which forces matter more, we turn to a quantitative exercise in the next section.

3.4 Extensions

The previous results are derived under a set of simplifying assumptions. In particular, we assumed that labor supply was fixed and that consumers were not able to share risk internationally. In this section, we consider how the previous results change when we relax those assumptions.

Endogenous labor supply We have derived the general structure under the assumption that consumers supply an amount of labor that is invariant to the state of the economy. However, when consumers change the amount of labor they supply in response to shocks, the equilibrium response of consumption and the real exchange rate will be different. A common way to introduce endogenous labor supply is to allow for an additive term in the utility function that captures the disutility of labor (e.g. Itskhoki & Mukhin (2021)). In Appendix D, we show that the effect of this type of endogenous labor supply enters through changing the terms-of-trade elasticity only. Therefore, Theorems 1 and 2 are unaffected. Moreover, the terms-of-trade elasticity is equal to the imports-to-consumption ratio in perfect competition.

[20]For some frictions, e.g. the introduction of markups in the manufacturing sector, we can determine what happens to the steady-state labor allocation to the final good sector, but not for all models.

International risk sharing So far, we have assumed that the small open economy was in a state of financial autarky. Hence, consumers were forced to consume their full income, stemming from wages, profits and net exports, in each period. Allowing for international risk sharing changes the equilibrium processes for final consumption and the real exchange rate from an AR(1) process to an ARMA(2,1) process. In this case, consumers save and dissave in response to domestic or foreign shocks which increases the persistence of the response to a similar size shock. Nevertheless, in the situation when the exogenous shocks approach a random walk, an adapted version of Theorem 2 still holds:

Theorem 3 (Terms-of-trade relative to TFP - International risk sharing)**.** *Under integrated and segmented financial markets with $\rho_y \to \infty$ with $y = \{D, X, M\}$, the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining the variance of final consumption is given by:*

$$\frac{\mathbb{V}\left(\Delta c_{St}|\varepsilon^{\$}_{Mt}, \varepsilon^{\$}_{Xt}\right)}{\mathbb{V}\left(\Delta c_{St}|\varepsilon_{Dt}, \varepsilon_{St}\right)} = \frac{\sigma^2_{\varepsilon,X}}{\sigma^2_{\varepsilon,A}} \frac{\left(\nu^m_c\left(H;\tilde{\Theta}\right)\right)^2}{\frac{\sigma^2_{\varepsilon,S}}{\sigma^2_{\varepsilon,D}} + \left(\frac{\mu-\nu^m_c(H;\tilde{\Theta})}{1-\gamma}\right)^2}$$

where $\sigma^2_{\varepsilon,i}$'s are the variances of the innovations to the shock processes.

Proof. See Appendix E. □

When the shocks approach random walks, the equilibrium process for consumption and the real exchange rate become ARIMA(1,1,1)-processes. Still, after applying the first difference operator, the resulting processes are stationary and Theorem 3 shows that the relative importance of terms-of-trade shocks relative productivity shocks in explaining consumption growth takes the same form as before. Importantly, we show that this result holds under integrated financial markets with non-state contingent local and foreign bonds and segmented financial markets. For this reason, Theorem 2 remains a useful limiting case in the presence of international risk sharing.

4 Quantitative exercise

In this section, we complement the qualitative comparison of the different models with a quantitative exercise. To this end, we calibrate the parameters in the model based on data from Chile and Colombia. In addition, by comparing the predictions of the model with moments taken from Colombian and Chilean firm-level trade data, we illustrate that the model with heterogeneous trade adjustment can generate the stylized facts of heterogeneous trade adjustment well. Finally, we leverage the rule of thumb described in Theorem 2 to compute the relative importance of TOT to TFP across the different models.

4.1 Calibration

Table 1 describes the calibrated parameters and their sources.

Table 1: Calibration of main parameters

Manufacturing sector		
Parameter	Value	Reference
γ	0.65	Country IO-tables
ω	0.50	Gopinath & Neiman (2014), Blaum et al. (2018)
ε	3.00	Gopinath & Neiman (2014), Blaum et al. (2018)
θ	3.00	Restriction
$\underline{\varphi}$	1.00	Melitz & Redding (2015)
κ	6.95	Estimation
f	0.05	Blaum et al. (2018)
Final good sector		
Parameter	Value	Reference
μ	0.40	Country IO-tables
σ	3.00	Gopinath & Neiman (2014), Blaum et al. (2018)

Input elasticities The input elasticity parameters γ and μ are calibrated to match the cost shares of the Chilean and Colombian manufacturing and the final good sectors, respectively. For the manufacturing input share, the Chilean data has values closer to 0.60, while the Colombian data has values closer to 0.70, so we pick a value in between to study a representative economy. There are no cross-country differences when it comes to μ, so we set it to 0.40. A third parameter that influences cost shares in the model is ω. This parameter cannot be easily matched to an observable moment in the data because we cannot separately identify the home-bias parameter from the relative price of domestic and imported intermediate input prices in equilibrium. Therefore, we follow Gopinath & Neiman (2014) and Blaum et al. (2018) and set ω to 0.50.

Elasticities of substitution The elasticity of substitution across final product varieties σ varies in the literature. Gopinath & Neiman (2014) uses a value of 4.00, while Blaum et al. (2018) uses the ratio of firms' revenues to total cost to back out the elasticity at the sectoral level. They find values in the range of 1.87 to 7.39, with most values in the 3.00 to 3.50 range. Kasahara & Rodrigue (2008) find values in the range of 3.14 to 4.44. We set σ to 3.00, which is in the range of estimates. The elasticity of substitution between imported and domestic inputs ε is also set to 4.00 in Gopinath & Neiman (2014) but Blaum et al. (2018) consider an estimate of 2.38. We set it to the intermediate value of 3.00. The elasticity of substitution between imported varieties θ is restricted to the same value as ε such that the model has an analytical solution as described in section 2. Below, we show that this restriction has no impact on replicating the main empirical facts.

Entry costs and debt elasticity The entry fixed cost is calibrated to 0.0075 in Gopinath & Neiman (2014), while it is calibrated to 0.0472 in Blaum et al. (2018). Given this substantial difference, we consider values between 0.005 and 0.05 but different values for f do not appear to change the quantitative results.

Targeting κ To calibrate the parameter that governs the degree of firm heterogeneity, κ, we rely on the fact that the model gives rise to an analytical expression for the distribution of firm-level imports conditional on importing.

Proposition 3. *Define aggregate imports in domestic currency* $M_t \equiv \int_{\varphi_{Mt}}^{\infty} P_{Mt}(\varphi)\, Q_{Mt}(\varphi)\, g(\varphi)\, d\varphi$, *then we have that:*

1. *The dollar amount imported by firm i,* $M_{it}^{\$}$, *can be written as the product of the fixed costs of importing and the firm-specific import measure:*

$$E_t M_t^{\$}(\varphi) = (\varepsilon - 1) W_t f \mathscr{L}_t(\varphi)$$

2. *The distribution of firm imports conditional on importing is Generalized Pareto:*

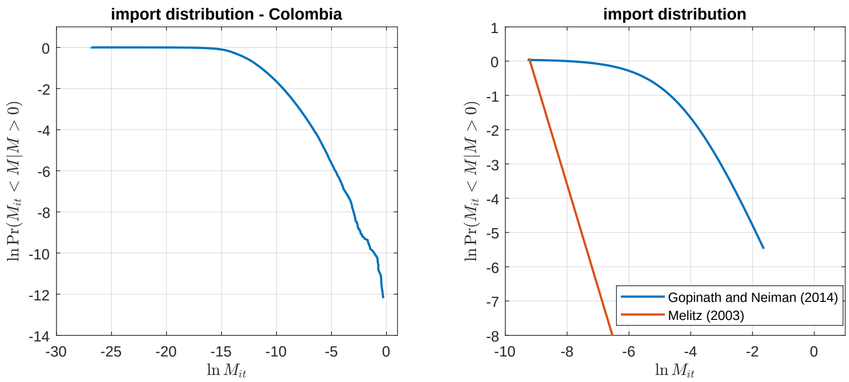
$$\Pr\left(M_{it}^{\$} < M | M > 0\right) = 1 - \left[1 + \frac{1}{\varepsilon - 1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon - 1} M\right]^{-\kappa\frac{\varepsilon - 1 - \gamma(\sigma - 1)}{(\sigma - 1)(\varepsilon - 1)}}$$

Proof. See Appendix F. □

The first part of Proposition 3 states that firm-level imports in domestic currency can be written as a combination of a term that is common for all firms times the number of intermediate input varieties sourced by the firm. Combining this intermediate result with the assumption that firm-level productivity follows a Pareto distribution, we obtain an expression for the distribution of imports across firms conditional on importing. In turn, we use Proposition 3 to calibrate κ by leveraging the fact that we now have an exact solution for what the tail exponent of the import distribution is. Combining the calibrated elasticities with the piecewise maximum-likelihood estimate of the tail exponent of the import distribution of the Colombian data, we arrive at an estimate for κ equal to 6.95.

Importantly, Figure 2 illustrates the importance of assuming that manufacturing firms pay a fixed cost per imported variety instead of assuming that firms pay simply one fixed cost to import, as in Melitz (2003). The left panel Figure 2 plots the relationship between log imports and the log of the cumulative distribution of imports in the Colombian data and illustrates the presence of many small importers and a few large importers. In panel (b) of Figure 2 we plot the same relationship for the two types of models. In a model where firms pay only one fixed cost to access imported intermediate inputs, the import distribution would follow a Pareto distribution and the relationship between the log import level and the log cumulative density of imports would be linear with slope $-\frac{\kappa}{\sigma - 1}$. However, when manufacturing firms have to incur a fixed cost per imported variety, the import distribution is Generalized Pareto with a much heavier tail. The model predicts a much

Figure 2: Power Law for Imports



Notes: The left-hand panel plots the log-log Pareto plot of the distribution of firm imports in Colombian data for the years 2006-2020. The right-hand panel plots the same log-log plot but of the model equilibrium following the expression in Proposition 3.

more important role for a few large importers and can generate the presence of many more small importers, which provides a much closer fit to the data.[21]

Equilibrium openness From Proposition (1) it follows that several variables jointly determine the equilibrium H without meaningfully entering into any of the relevant elasticities that make up the dynamic system. For example, in the perfect and monopolistic competition cases, it follows that $(LP_M^{\$})/(P_X^{\$}X)$ jointly determine H, so we don't need to take a stand on the particular values of foreign prices, the export quantity and the labor force in levels. In practice, we use Colombian and Chilean national accounts to calibrate H as the ratio of total imports to total household consumption together with the calibrated input shares $\hat{H} = T^{-1}\sum(1/\mu\gamma)(P_M M)/(PC)$. In Colombia, this average is 0.87 and is calculated using quarterly data covering the years 2006-2020 while in Chile this average is 0.93 for the year 2008-2018, which we use to target an H of 0.90 in our calibration.

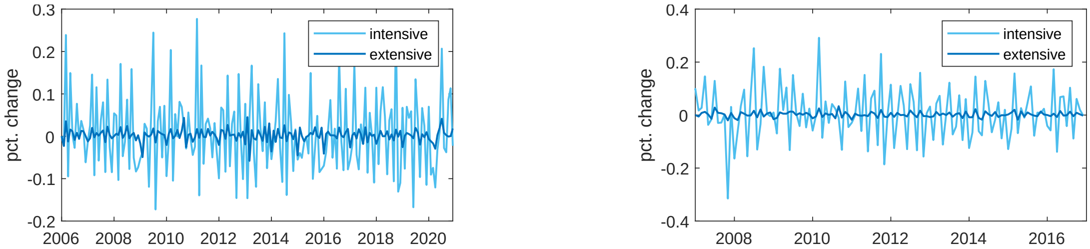
4.2 Moments of trade adjustment

To achieve tractability we assumed that fixed costs per variety increase linearly with the number of imported varieties and that the elasticity of substitution between imported varieties θ is the same as the elasticity of substitution between the imported and domestic intermediate input bundles ε. Before turning to the quantitative exercise, we now show that these simplifying assumptions do not compromise the model's ability to replicate stylized facts of heterogeneous trade adjustment. Besides generating a distribution of firm-level imports conditional on importing that is close to the data, the model also predicts that (1) the firm-intensive margin dominates the firm-extensive margin, (2) the importance of the sub-intensive margin increases with firm size, and (3) terms-of-trade shocks generate endogenous TFP movements.

[21] It turns out the number of small importers in the data is even higher than the complete can generate. As discussed in Arkolakis (2010) modeling fixed costs as market penetration costs could potentially generate more small importers.

Figure 3: Trade Adjustment Margins

(a) Colombia **(b)** Colombia



Notes: These figures plot the percentage changes in firm-intensive and firm-extensive margins at the quarterly frequency for Colombia in panel (a) and for Chile in panel (b). For Colombia, we include trade volumes net of oil. This excludes the following HS-4 codes: 2709-15, 3403, 3819, 3811 and 3911. For Chile, we include the volumes net of copper. This excluded the following HS-4 codes: 2603, 2825, 2827, and all items under HS-2 74.

Firm-intensive versus firm-extensive margin The total change in imports can be decomposed into a firm-intensive margin and a firm-extensive margin. The firm-intensive margin measures the change in overall imports that is due to continuing importers changing firm-level imports. The firm-extensive margin captures changes in overall imports as firms start and stop importing altogether. Formally,

$$\underbrace{\frac{\Delta m_t}{m_{t-1}}}_{\Delta\text{ Aggregate Imports}} = \underbrace{\sum_{i\in\Omega_t^f\cap\Omega_{t-1}^f}\frac{m_{it}-m_{it-1}}{m_{t-1}}}_{\text{Firm-intensive margin}} + \underbrace{\sum_{i\in\Omega_t^f\setminus\Omega_{t-1}^f}\frac{m_{it}}{m_{t-1}} - \sum_{i\in\Omega_{t-1}\setminus\Omega_t^f}\frac{m_{it-1}}{m_{t-1}}}_{\text{Firm-extensive margin}}$$

Figure 3 plots the split of aggregate change in import values into a firm-intensive margin and a firm-extensive margin for Colombia and Chile separately. For both countries, changes at the firm-intensive margin dominate changes at the extensive firm margin.

The prediction that the importance of the firm-extensive margin in explaining changes in aggregate trade is small, is also borne out in the model. In particular,

Proposition 4 (Firm-intensive and firm-extensive margin)**.** *Changes in aggregate imports are given by:*

$$-\frac{\partial \ln M_t}{\partial \ln x_t} = -\frac{x_t}{M_t}\left[\underbrace{\int_{\varphi_{Mt}}^{\infty}\frac{\partial}{\partial x_t}\tilde{M}_t\mathcal{L}_t(\varphi)dG(\varphi)}_{\text{Intensive}} - \underbrace{\tilde{M}_t\mathcal{L}_t(\varphi_{Mt})\frac{\partial}{\partial x_t}\varphi_{Mt}}_{\text{Extensive}}\right]$$

Following any infinitesimal aggregate shock, changes in aggregate imports are accounted for by the firm-intensive margin of trade only.

Proof. See Appendix F. □

Proposition 4 guarantees that all of the adjustments in aggregate imports happen at the intensive margin, which is the case in the data. With no heterogeneity, this is true by construction. However, in the model with heterogeneity and selection, the same is true because the contribution of the extensive margin depends on the measure of imported intermediate inputs which is zero when evaluated at the cut-off productivity level.

Firm sub-intensive versus firm sub-extensive margin Following Gopinath & Neiman (2014), we further decompose the firm-intensive level margin into a firm sub-intensive and a firm sub-extensive margin as follows:

$$\begin{aligned} \underbrace{\frac{\Delta m_t}{m_{t-1}}}_{\text{Total change in imports}} &= \underbrace{\sum_{i\in\Omega_t^f\setminus\Omega_{t-1}^f}\frac{m_{it}}{m_{t-1}} - \sum_{i\in\Omega_{t-1}\setminus\Omega_t^f}\frac{m_{it-1}}{m_{t-1}}}_{\text{firm-extensive margin}} \\ &+ \underbrace{\sum_{i\in\Omega_t^f\cap\Omega_{t-1}^f}\left[\underbrace{\sum_{j\in\Omega_{it}^p\setminus\Omega_{it-1}^p}\frac{m_{ijt}}{m_{it-1}} - \sum_{j\in\Omega_{it-1}^p\setminus\Omega_{it}^p}\frac{m_{ijt-1}}{m_{it-1}}}_{\text{firm sub-extensive margin}} + \underbrace{\sum_{j\in\Omega_{it}^p\cap\Omega_{it-1}^p}\frac{m_{ijt}-m_{ijt-1}}{m_{t-1}}}_{\text{firm sub-intensive margin}}\right]}_{\text{firm-intensive margin}} \end{aligned} \tag{4.1}$$

where Ω_t^f is the set of firms importing in period t, Ω_{it}^p is the set of products imported by firm i at time t and m_{ijt} is the imported volume of product j by firm i. The firm sub-intensive margin captures the extent to which firms change firm-level imports by importing a different amount of the set of varieties they already import, while the firm sub-extensive margin measures the extent to which firms change firm-level imports by changing the set of varieties being imported. Figure 4 indicates that the firm sub-intensive and firm sub-extensive margins each explain around 50% of the firm-intensive margin in both countries. More importantly, the relative importance of the sub-intensive versus the sub-extensive margins differs greatly across the firm-size distribution. To illustrate this, Figure 5 shows the importance of the firm sub-intensive margin as a share of the firm-intensive margin for different firm-size percentiles. As we move from the lower end of the firm-size distribution to the upper tail of the firm-size distribution, the importance of the firm sub-intensive margin increases, but it turns out that even the largest importers adjust both on the firm sub-intensive and firm sub-extensive margin.

The model also generates a positive relationship between firm size and the importance of the firm-sub-intensive margin in response to a commodity price shock.

Proposition 5 (Firm sub-intensive vs firm sub-extensive margin)**.** *Conditional on a commodity price shock $p_{Xt}^{\$}$, the model with heterogeneous firms predicts that the share of the sub-intensive margin relative to the overall change to total dollar-imports per firm is given by*

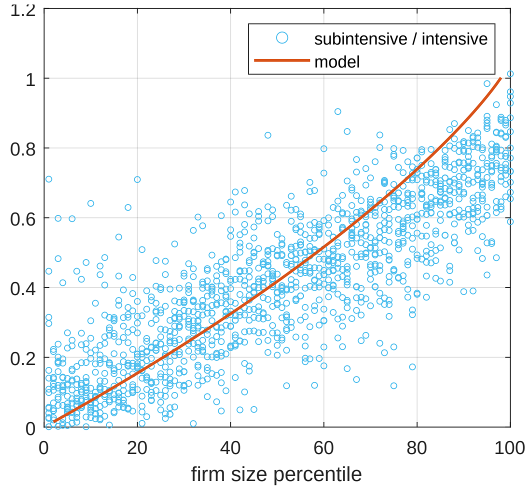
$$\frac{\frac{\mu}{1-\mu}\nu_{pH}-\nu_{qH}}{\frac{\mu}{1-\mu}\nu_{pH}-\nu_{qH}+(\varepsilon-1)\left(\nu_{pH}+\nu_{qH}+\frac{1}{1-\gamma_{Di}}\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\frac{(\kappa-(\sigma-1))^{-1}}{(1-\tilde{\kappa}H)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+(1-H)\frac{\sigma-1}{\sigma}}\right)}$$

Figure 4: Trade Adjustment Margins

(a) Colombia **(b)** Colombia

Notes: These figures plot the percentage changes in the firm sub-intensive and firm sub-extensive margin at the quarterly frequency for Colombia in panel (a) and for Chile in panel (b). For Colombia, we include trade volumes net of oil. This excludes the following HS-4 codes: 2709-15, 3403, 3819, 3811 and 3911. For Chile, we include the volumes net of copper. This excluded the following HS-4 codes: 2603, 2825, 2827, and all items under HS-2 74.

Figure 5: Sub-Intensive vs. Sub-Extensive Margin



Notes: The figure plots the relationship between the level of imports and the share attributed to the sub-intensive margin observed in the data and predicted by the model in the baseline calibration. The theoretical relationship is obtained by noting that Proposition 3 allows us to solve for any percentile of the distribution and its associated level of imports. Consequently, we can map any percentile to a productivity level $\varphi_p = (1-p)^{-\frac{1}{\kappa}}\varphi_{Mt}$ and each productivity level in turn to its domestic input share γ_{Dp}, which, are finally used to map import size percentiles to their associated sub-intensive margin shares.

and is decreasing in the domestic input share γ_{Di}.

Proof. See Appendix F. □

In proposition 5, we focus on commodity price shocks for two reasons. First, section **??** indicates that these shocks account for most movements in the terms-of-trade. Second, in response to a commodity price, the change in the firm-sub-intensive and firm-sub-extensive margins has the same sign which makes the ratio of the two margins interpretable as shares.[22] Figure 5 shows that the model closely matches the slope in the data even though this moment is not targeted in the calibration.

Manufacturing TFP Finally, there are several papers that present evidence of changes in aggregate productivity through reallocation across firms in response to terms-of-trade shocks (e.g. Pavcnik (2002) and Halpern et al. (2015)). The model with heterogeneous trade adjustment is capable of generating endogenous movements in manufacturing TFP in response to terms of trade and interest rate shocks as well.

Proposition 6 (The need for selection)**.** *Across all models,*

1. *the aggregate production function in the manufacturing sector is given by:*

$$Y_{Dt} = \underbrace{A_{Dt}}_{\textit{Technology}} \underbrace{{L_{Dt}}^{1-\gamma} {X_{Dt}}^{1-\gamma}}_{\textit{Factor use}} \underbrace{\left[\int_{\underline{\varphi}}^{\infty} \left(\varphi_i \left(\frac{L_{Dt}(\varphi)}{L_{Dt}} \right)^{1-\gamma} \left(\frac{X_{Dt}(\varphi)}{X_{Dt}} \right)^{\gamma} \right)^{\frac{\sigma-1}{\sigma}} d(\varphi) \right]^{\frac{\sigma}{\sigma-1}}}_{\textit{Allocative efficiency}}$$

2. *In the absence of selection, the heterogeneous manufacturing sector can be replaced by a representative producer with the following productivity level.*

$$\varphi_D = \underline{\varphi} \left(\frac{\kappa}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

Proof. See Appendix F. □

The aggregate production function in the manufacturing section is composed of three elements that map into the framework of Baqaee & Farhi (2020). The first term is the *technology* term, namely exogenous productivity in the manufacturing sector. The second term captures the contribution of *input and factor use* to output. L_{Dt} accounts for productive labor in manufacturing and X_{Dt} is an intermediate input aggregator that accounts for total input use.[23] Finally, the *allocative efficiency*

[22] In response to productivity shocks and import price shocks, the margins move in opposite directions. This makes defining the share attributed to a particular margin non-trivial.

[23] Even though the model has inelastic total labor supply, an increase in productive labor can happen at the expense of a reduction in labor used in importing.

term represents the reallocation of productive labor and inputs between firms. Whenever very productive firms (high φ) are allocated more labor and inputs, output increases above and beyond the increase in aggregate labor and inputs allocated to the manufacturing sector.

In the absence of changes in the productivity cut-off for importing the model collapses back into a representative-producer framework in which terms-of-trade shocks do not have first-order effects on aggregate productivity as in Kehoe & Ruhl (2008). It follows that heterogeneity, fixed costs, and roundabout production are necessary, but not sufficient conditions for terms-of-trade shocks to induce aggregate productivity effects. Instead, in this model, endogenous selection into importing is also necessary to generate endogenous movements in TFP.[24]

4.3 Quantitative importance of terms-of-trade shocks

After establishing that the model with heterogeneous trade adjustment captures key facts of heterogeneous trade adjustment, we turn to quantitatively evaluating each model's prediction regarding the relative importance of terms-of-trade shocks. More specifically, theorem 2 enables us to estimate the relative importance of terms of trade shocks to total factor productivity shocks for any ratio of their variances. Regardless of the specific values for these variances, we can determine the relative impact of TOT shocks compared to productivity shocks in the models we consider.

We present the results from the rule-of-thumb exercise in Table 2. We have two main takeaways. First, conditional on the structural parameters Θ, the last column of Panel A of Table 2 shows that the benchmark SOE-ITBC model understates the importance of the terms of trade by a factor of two to five when compared to a model with heterogeneous trade adjustment, depending on whether we consider the upper or lower bound of the shares. Comparing across the models, thirty-four percent of the gap is explained by moving from a benchmark SOE-IRBC economy to an economy in which the manufacturing sector competes under monopolistic competition. An additional sixty-two is explained by introducing increasing returns to importing. The inclusion of a selection mechanism in the model makes up for the remaining four percent. According to Proposition 2, these differences either originate from differences in the imports-to-consumption ratio or from differences in the distortion term. To this end, Table 2 presents the steady-state level of trade openness, H^m, and the size of the distortion term, $\Lambda^m(H,\Theta)$ alongside the value of the terms-of-trade elasticity, $\nu_c\left(H,\tilde{\Theta}\right)$. In addition, we provide the ratio of steady-state trade openness in each of the models to the one in the benchmark SOE-IRBC model and the ratio of each model's terms-of-trade elasticity to the terms-of-trade elasticity in the benchmark SOE-IRBC model.[25] From comparing these relative quantities, it is clear that the main driver of the differences in the models originates from the

[24] This result is akin to Blaum et al. (2018) which shows that the percentage change in the domestic input share is a sufficient statistic to measure the aggregate gains from input trade. In our model, the change in the domestic share is log-linear in the change in the productivity cut-off. Hence, without selection, there is no difference in the change in the aggregate domestic input share between a representative-firm model with roundabout production and a heterogeneous firms model with fixed costs of importing and roundabout production.

[25] Because the imports-to-consumption ratio is given by $\mu\gamma H^m(\Theta)$, comparing the levels of steady-state trade openness is sufficient to compare the imports-to-consumption ratios across the models.

Table 2: TOT relative to TFP

Model	H^m	H^m/H^{IRBC}	$\Lambda^m(H,\Theta)$	$\nu_c^m\left(H,\tilde{\Theta}\right)$	$\frac{\nu_c^m(H,\tilde{\Theta})}{\nu_c^{\text{IRBC}}(H,\tilde{\Theta})}$	$\frac{\mathbb{V}\left(c_{St}\mid p^{\$}_{Mt},p^{\$}_{Xt}\right)}{\mathbb{V}(c_{St}\mid a_{Dt},p_{St})}$
PANEL A: CONDITIONAL ON Θ						
SOE-IRBC	0.652	-	1	0.1695	-	[0.0201; 0.0662]
MC	0.794	1.217	0.967	0.1995	1.177	[0.0300; 0.121]
IRS	0.926	1.420	0.997	0.2401	1.417	[0.0477; 0.276]
HTA	0.929	1.425	1.004	0.2425	1.431	[0.0489; 0.290]
PANEL B: CONDITIONAL ON $H^m(\Theta)$						
SOE-IRBC	0.929	-	1	0.2416	-	[0.0484; 0.285]
MC	0.929	1	0.990	0.2393	0.990	[0.0473; 0.271]
IRS	0.929	1	0.997	0.2409	0.997	[0.0481; 0.281]
HTA	0.929	1	1.004	0.2425	1.004	[0.0489; 0.290]

Notes: This table considers the two quantitative exercises we consider. The panel "conditional on Θ" shows considers the experiment in which we keep the set of structural parameters fixed and allow changes in the value of ν_c both because the expression differs across the models and because the trade openness changes. In the panel "conditional on $H^m(\Theta)$", we ensure that all models generate the same level of steady-state trade openness. For each experiment, we provide the corresponding value of trade openness H, the value of the distortion, $\Lambda^m()$, the value for the general equilibrium elasticity, $\nu_c\left(H,\tilde{\Theta}\right)$, and the upper and lower bound on the relative importance of terms-of-trade shocks in explaining consumption volatility, given by

$$\frac{\mathbb{V}\left(c_{St}\mid p^{\$}_{Mt},p^{\$}_{Xt}\right)}{\mathbb{V}\left(c_{St}\mid a_{Dt},p_{St}\right)}=\frac{\sigma_X^2}{\sigma_A^2}\frac{\left(\nu_c\left(H,\tilde{\Theta}\right)\right)^2}{\frac{\sigma_S^2}{\sigma_D^2}+\left(\frac{\mu-\nu_c(H,\tilde{\Theta})}{1-\gamma}\right)^2}.$$

The upper and lower bound correspond to the cases where σ_S^2/σ_D^2 are one and zero, respectively. We assume only consider cases in which productivity in the final good sector is less volatile or equally volatile than in manufacturing. "SOE-IRBC" refers to the benchmark SOE-IRBC model, "MC" refers to the monopolistic competition model, "IRS" refers to the model with increasing returns to importing, and "HTA" refers to the model with heterogeneous trade adjustment.

imports-to-consumption ratio and that the distortion term only plays a secondary role. In other words, most of the difference can be attributed to the equilibrium values of trade openness.

Second, the importance of steady-state trade openness also suggests that the predictions of the different models regarding the importance of terms-of-trade shocks relative to productivity shocks in explaining consumption volatility might be reduced if we were to ensure that each model predicts the same level of steady-state level of trade openness. To examine this, we redo the quantitative exercise and ensure that all models are calibrated to generate the state-state openness level of the complete model. We implement this by appropriately changing the home-bias parameter.[26] Indeed, when we compare the predictions regarding the relative importance of terms-of-trade shocks in explaining consumption volatility in panel B, the differences across the models essentially vanish. In this case, neither does the imports-to-consumption ratio vary, which is by construction, nor does the distortion term quantitatively vary across the models. We take this as evidence that as long as one appropriately targets steady-state trade openness, introducing distortions to account for micro-moments of trade adjustment does not meaningfully affect the relative importance of

[26]The home-bias parameter is suitable because it co-determines the choice between domestic and imported intermediate inputs and does not enter the general equilibrium elasticities. Therefore, changing the home-bias parameter only affects the steady-state allocations and not the dynamic properties of the model.

terms-of-trade shocks in the models we consider.

5 Conclusion

In this paper, we examine whether accounting for heterogeneous trade adjustment across firms in a benchmark SOE-IRBC model changes the importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining consumption volatility in commodity-exporting countries. We develop a small open economy model in which the country exports an endowment stream of commodities, imports intermediate inputs to be used in producing manufacturing output, and produces the final good in a downstream sector. Domestic manufacturing producers, with varying productivity levels, self-select into importing but must pay a fixed cost for each imported intermediate input variety. This results in an equilibrium where more productive domestic manufacturing producers are more susceptible to exchange rate fluctuations and adjust by adjusting both on the firm-sub-intensive and firm-sub-extensive margins. We demonstrate that the model encompasses simpler cases found in the literature, including standard SOE-IRBC models, models with homogeneous firms competing under monopolistic competition, and models with increasing returns to importing.

We show that the equilibria of the benchmark SOE-IRBC model and the model with heterogeneous trade adjustment and all the models in between can be represented by one non-linear equation in one endogenous variable, the economy's trade openness as it captures the extent to which production of final consumption depends on imported intermediate inputs. We show that, in the steady-state equilibrium, the added frictions lead to a more open economy, as manufacturing producers try to avoid double marginalization at home and increase imports in response. In addition, up to a first-order, the dynamics of the models can be summarized in a common structure. In particular, changes in consumption and changes in the real exchange rate in response to changes in openness are captured by two partial-equilibrium elasticities whose values depend on the particular model. Moreover, the same two partial equilibrium elasticities collectively make up the terms-of-trade elasticity that controls the relative importance of terms-of-trade shocks compared to TFP shocks in explaining consumption volatility across the models.

To understand whether models that account for micro-moments of heterogeneous trade adjustment across firms have different predictions for the relative importance of terms-of-trade shocks, we conduct two experiments. Conditional on the structural parameters of the model, we find that considering these micro-moments increases the significance of terms-of-trade shocks by a factor of two to five. This difference is mostly explained by the introduction of monopolistic competition and increasing returns to importing which increases the incentives to import. While the introduction of heterogeneity and selection is essential to capture micro-moments of heterogeneous trade adjustment, it does not meaningfully change the relative importance of terms-of-trade shocks relative to a model with monopolistic competition and increasing returns to importing. Conditional on the steady-state trade openness of the economy, the different models attribute roughly an equal importance to terms-of-trade shocks in explaining consumption volatility. Taken together, these two experiments imply that the introduction of frictions to account for realistic firm-level trade adjustment only has a limited impact on the model's ability to generate consumption volatility from terms-of-trade shocks. This is because a benchmark SOE-IRBC model calibrated to the same steady-state trade openness generates the same relative importance of terms-of-trade shocks relative to sectoral TFP shocks in explaining consumption volatility compared to a model with heterogeneous trade adjustment.

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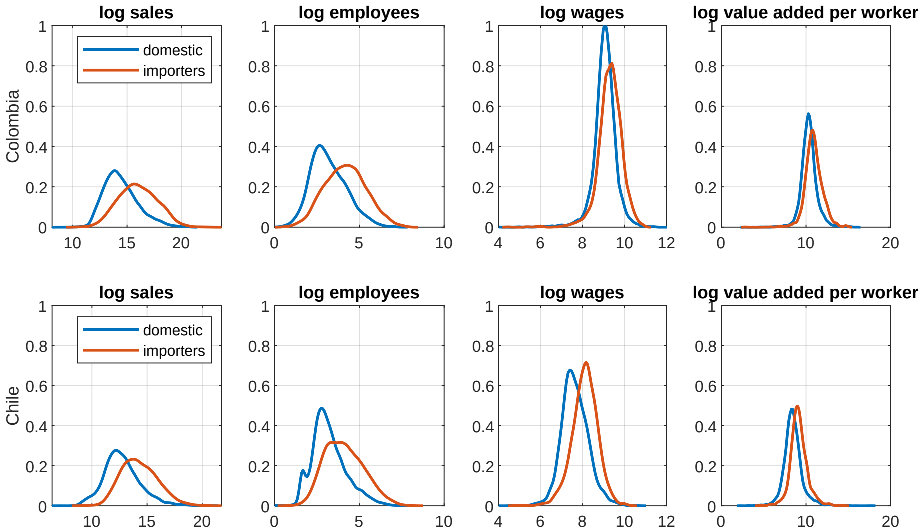
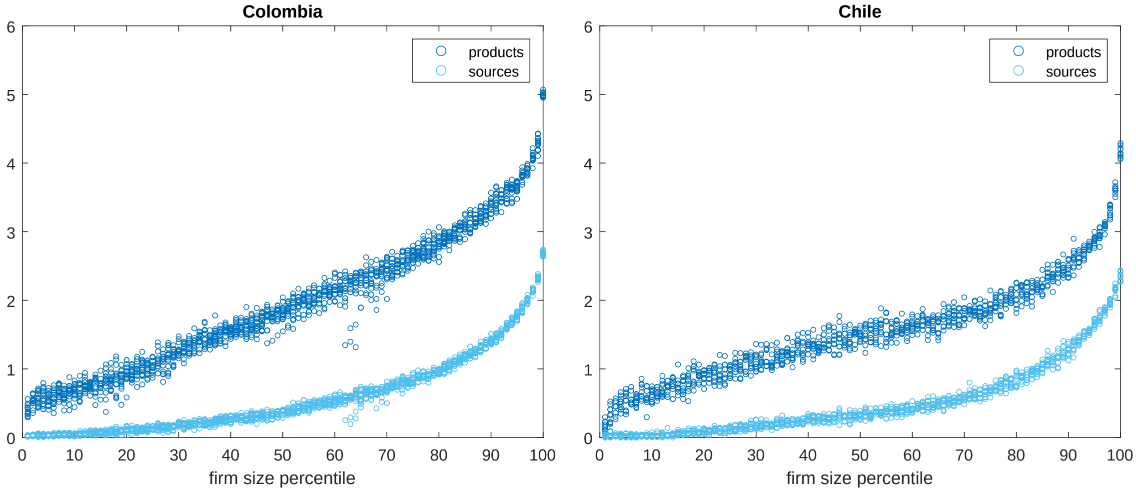
A Descriptive statistics

Figure A.1: Import premia



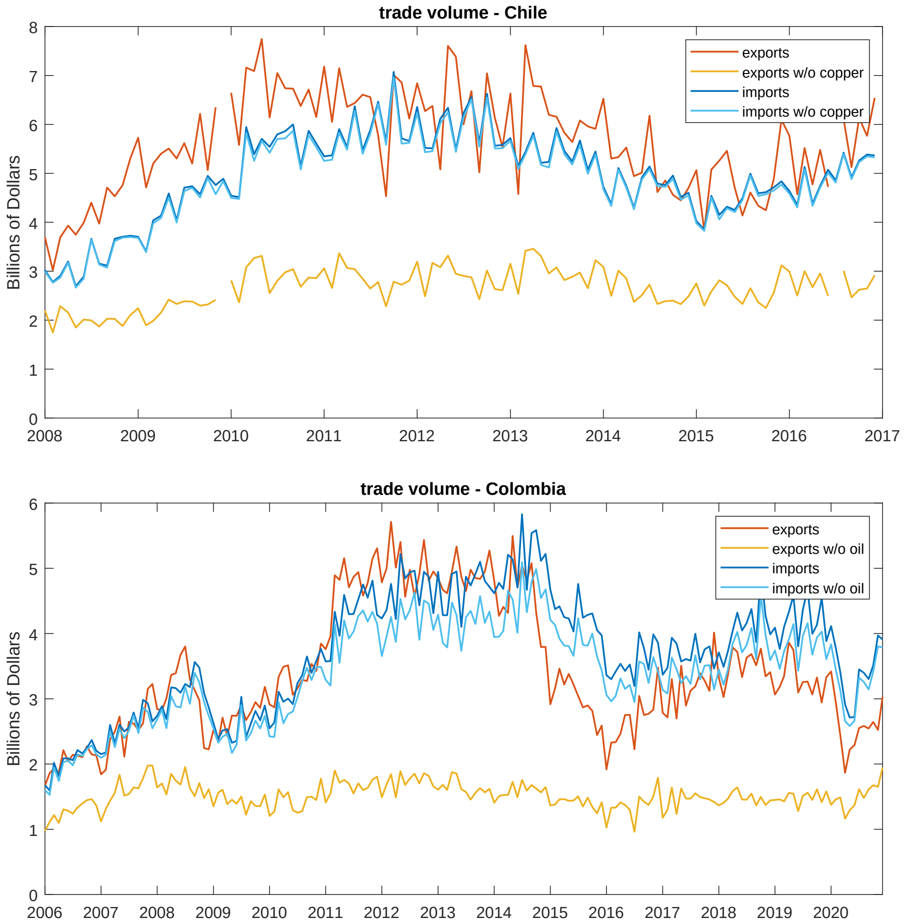
Notes: Kernel densities of log sales, number of employees, wages, and value-added per worker. Includes only firms that either are exclusively participating in the domestic market, that is, firms that do not import or export, and firms that are importers only.

Figure A.2: Number of products and source across firms



Notes: This figure plots the log number of products and sources by import size percentile.

Figure A.3: Aggregate Trade Flows



Notes: Trade volumes in current US dollars. The volumes net of oil excludes the following HS-4 codes: 2709-15, 3403, 3819, 3811 and 3911. The volumes net of copper exclude the following HS-4 codes: 2603, 2825, 2827, and all items under HS-2 74.

Table A.1: Time series properties of aggregates

	Chile		Colombia		Chile			Colombia		
	σ_{x_{t-1},x_t}	σ_x	σ_{x_{t-1},x_t}	σ_x		σ_{xy}			σ_{xy}	
y_t	0.619	2.160	0.556	2.679	1.000			1.000		
c_t	0.507	3.913	0.578	3.010	0.884	1.000		0.914	1.000	
tb_t					-0.197	-0.156	1.000	0.039	0.050	1.000
q_t		0.020		0.025						
s_t		0.044		0.042						

Notes: Relative standard deviations, AR(1) persistence and correlations between output, consumption, and the trade balance. Data is quarterly and covers the year 2005-2022 for Colombia and 1996-2021 for Chile. The trade balance is computed as exports minus imports over GDP.

B Non-linear solutions

B.1 Final goods sector

The final goods sector is made up of homogeneous producers that combine labor (L_{St}) with the final manufacturing output (Y_{St}) to produce the final consumption good Y_{St}. They have access to the following technology:

$$Y_{St} = A_{St} L_{St}^{1-\mu} X_{St}^{\mu}$$

Services producers solve the following cost minimization problem:

$$\begin{aligned} &\min_{L_{St}, X_{St}} W_t L_{St} + P_{Dt} X_{St} \\ &\text{s.t.} \qquad Y_{St} = A_{St} L_{St}^{1-\mu} X_{St}^{\mu} \end{aligned}$$

This yields the following first-order conditions

$$W_t L_{St} = (1-\mu)\mathrm{MC}_{St} Y_{St} \quad \text{and} \quad P_{Dt} X_{St} = \mu \mathrm{MC}_{St} Y_{St}$$

and the following marginal cost function:

$$\mathrm{MC}_{St} = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}}$$

Because services producers compete in a perfectly competitive manner, they price to marginal cost. Therefore the price of services is given by:

$$\mathrm{P}_{St} = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}} \tag{B.1}$$

B.2 Manufacturing sector

The equilibrium manufacturing price index depends on the assumed production structure. We consider four options: (1) Homogeneous firms that compete under perfect competition and do not have access to an increasing returns to scale importing technology, (2) Homogeneous firms that compete under monopolistic competition and do not have access to an increasing returns to scale importing technology, (3) Homogeneous firms that compete under monopolistic competition and have access to an increasing returns to scale importing technology, (4) heterogeneous firms that compete under monopolistic competition and that can self-select into an increasing returns to scale importing technology.

B.2.1 Homogeneous firms under perfect competition

In this section, we provide the derivations for the model where domestic manufacturers are Homogeneous in their productivity and where the importing technology is not subject to economies of scale. Manufacturers compete under monopolistic competition and have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \quad \text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Optimal conditional input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned} \min_{L_{Dit}, X_{Dit}} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\ \text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\ & X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

The first-order conditions for the cost minimization problem are the following. In the upper tier, manufacturing firms choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xit}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\text{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dit} = \gamma \text{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs (Q_{Dit}, Q_{Mit}) given inputs prices P_{Dt} and $E_t P_{Mt}^{\$}$, both denominated in domestic currency. The first-order conditions from the second-tier problem are given by:

$$P_{Dt} Q_{Dt} = \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon-1} P_{Xt} X_{Dt} \quad \text{and} \quad E_t p_{Mt}^{\$} Q_{Mit} = (1-\omega)\left(\frac{P_{Xit}}{E_t p_{Mt}^{\$}}\right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

Given that these prices do not depend on the identity of the firm, we can drop the i subscript and combine them to write the marginal cost function as:

$$\text{MC}_{Dt} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xt} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) E_t {p_{Mt}^{\$}}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Manufacturing price index Combining the fact that $P_{Dit} = \text{MC}_{Dit}$ the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$
\begin{aligned}
P_{Dt} &\equiv \left(\int_i P_{Dit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di \\
&= \left(\int_i (\mathrm{MC}_{Dit})^{1-\sigma}\, di\right)^{\frac{1}{1-\sigma}} \\
&= \left(\int_i \left(\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mit}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \qquad \text{(B.2)} \\
&= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}
\end{aligned}
$$

B.2.2 Homogeneous firms under monopolistic competition

In this section, we provide the derivations for the model where domestic manufacturers are homogeneous in their productivity and where the importing technology is not subject to economies of scale. Manufacturers compete under monopolistic competition and have access to the following technology:

$$
Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \quad \text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
$$

Optimal conditional input allocation They solve a two-tiered cost minimization problem:

$$
\begin{aligned}
\min_{L_{Dit}, X_{Dit}} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\
\text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\
& X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

The first-order conditions for the cost minimization problem are the following. In the upper tier, manufacturers choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xit}. The first-order conditions are given:

$$
W_t L_{Dit} = (1-\gamma)\mathrm{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dit} = \gamma \mathrm{MC}_{Dit} Y_{Dit}
$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs (Q_{Dit}, Q_{Mit}) given inputs prices P_{Dt} and $E_t P_{Mt}^{\$}$, both denominated in domestic currency.

The first-order conditions from the second-tier problem are given by:

$$P_{Dt}Q_{Dt} = \omega\left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon-1} P_{Xt}X_{Dt} \quad \text{and} \quad E_t p^{\$}_{Mt} Q_{Mit} = (1-\omega)\left(\frac{P_{Xit}}{E_t p^{\$}_{Mt}}\right)^{\varepsilon-1} P_{Xit}X_{Dit}$$

Given that these prices do not depend on the identity of the firm, we can drop the i subscript and combine them to write the marginal cost function as:

$$\text{MC}_{Dt} = \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Xt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xt} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega)E_t {p^{\$}_{Mt}}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Manufacturing price index Combining the fact that $P_{Dt} = \frac{\sigma}{\sigma-1}\text{MC}_{Dt}$, the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$\begin{aligned}
P_{Dt} &\equiv \left(\int_i P_{Dit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di \\
&= \left(\int_i \left(\frac{\sigma}{\sigma-1}\text{MC}_{Dit}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\int_i \left(\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega){P_{Mit}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}
\end{aligned} \tag{B.3}$$

B.2.3 Homogeneous firms under monopolistic competition and IRS importing

In this section, we provide the derivations for the model where domestic manufacturers are Homogeneous in their productivity and where the importing technology is subject to economies of scale. Manufacturers have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dt}^{1-\gamma} X_{Dit}^{\gamma}$$

$$\text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad Q_{Mit} = \left(\int_{k\in|\mathscr{L}_{it}|} {q_{Mkit}}^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}}$$

The optimal production strategy is determined in two steps. First, conditional on the sourcing strategy $|\mathscr{L}_{it}|$, manufacturers choose the cost-minimizing bundle of labor and intermediate inputs and the cost-minimizing bundle of domestic and foreign intermediate inputs for each level of output. Second, given this production structure manufacturers determine the optimal measure $|\mathscr{L}_{it}|$ of imported intermediate input varieties subject to the fixed costs of importing.

Optimal conditional input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned}
\min_{L_{Dit},X_{Dt}\big| |\mathscr{L}_{it}|} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\
\text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\
& X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}\left(|\mathscr{L}_{it}|\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}$$

The first-order conditions for the cost minimization problem are the following. In the upper tier, manufacturers choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xt}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\text{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dt} = \gamma \text{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs $(Q_{Dit}, Q_{Mit}(|\mathscr{L}_{it}|))$ given inputs prices P_{Dt} and $P_{Mit}(|\mathscr{L}_{it}|)$, both denominated in domestic currency. The first-order conditions from the second-tier problem are given by:

$$P_{Dt} Q_{Dit} = \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon-1} P_{Xit} X_{iDt} \quad \text{and} \quad P_{Mit} Q_{Mit} = (1-\omega) \left(\frac{P_{Xit}}{P_{Mit}(|\mathscr{L}_{it}|)} \right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

These first-order conditions can be combined to write the marginal cost function as:

$$\text{MC}_{Dit} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xit} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) P_{Mit}(|\mathscr{L}_{it}|)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

Sourcing strategy Given the optimal production structure conditional on the sourcing strategy, we now solve for the optimal sourcing strategy assuming that firms choose the sourcing strategy that maximizes their profits:

$$\begin{aligned}
\max_{|\mathscr{L}_{it}|} & \left(P_{Dit} - MC_{Dit} \right) Y_{it} - W_t f |\mathscr{L}_{it}| \\
\text{s.t.} \quad & MC_{Dit} = \frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \\
& P_{Xit} = \left[\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) \left(E_t P^{\$}_{Mt} \right)^{1-\varepsilon} |\mathscr{L}_{it}| \right]^{\frac{1}{1-\varepsilon}} \\
& Y_{Dt} = \left(\frac{P_{Dit}}{P_{Dt}} \right)^{-\sigma} \left(X_{St} + Q_{Dt} \right) \\
& P_{Dit} = \frac{\sigma}{\sigma-1} MC_{Dit}
\end{aligned}$$

where we have used the assumption that $\theta = \epsilon$ such that $P_{Mt} = E_t p^{\$}_{Mt} |\mathscr{L}_{it}|^{\frac{1}{\epsilon-1}}$ or when all constraints are substituted in

$$\max_{|\mathscr{L}_{it}|} \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P^{\sigma}_{Dt}(X_{St}+Q_{Dt})\cdot \left[\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}|\mathscr{L}_{it}|\right)^{\frac{\gamma}{1-\varepsilon}}\right]^{1-\sigma} - W_t f |\mathscr{L}_{it}|$$

Now we propose a change of variables in the maximization problem. Let

$$Z_t = \left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}|\mathscr{L}_{it}|\right)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \Rightarrow |\mathscr{L}_{it}| = \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega {P_{Dt}}^{1-\varepsilon}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}}$$

such that the maximization problem becomes

$$\max_{Z_t} \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} {P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})\left[\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right]^{1-\sigma} Z_t - W_t f \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega {P_{Dt}}^{1-\varepsilon}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}}$$

The first-order condition of this problem is the following.

$$\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} {P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})\left[\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right]^{1-\sigma} - W_t f \frac{\varepsilon-1}{\gamma(\sigma-1)}\frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}-1}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}} = 0$$

Hence we have an expression for Z_t:

$$Z_t^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\gamma(\sigma-1)}} = \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{\gamma(\sigma-1)}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}(1-\omega)\cdot \left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}\left[\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right]^{1-\sigma}$$

and consequently

$$\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}|\mathscr{L}_{it}|\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}} = \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{\gamma}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}\left[\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right]^{1-\sigma}$$

We can then solve for the measure of imported varieties.

$$|\mathscr{L}_{it}| = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\left(\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{W_t{}^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\right]^{\frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon}$$

This expression does not depend on the identity of the firm and therefore all firms have the same sourcing strategy. At the same time, this expression defines the minimal level of productivity φ_D necessary for firms to import and to cover the fixed costs. This is found at $|\mathscr{L}_{it}|(\varphi_{Mt}) = 0$:

$$\varphi_{Mt} = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}} \cdot \frac{1}{A_{Dt}}\frac{W_t{}^{1-\gamma}E_t P_{Mt}^{\$}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

We can use this expression to write the measure of imported inputs more succinctly as a function of the importing cutoff where we drop the subscript i:

$$|\mathscr{L}_t| = \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon}\left[\left(\frac{\varphi_D}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]$$

We can then use this result to solve for firm-specific input prices and unit costs, respectively. We have that

$$P_{Xt} = \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{1}{1-\varepsilon}}P_{Dt}$$

Manufacturing price index Combining the fact that $P_{Dit} = \frac{\sigma}{\sigma-1}\text{MC}_{Dit}$, the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$
\begin{aligned}
P_{Dit} &\equiv \left(\int_i P_{Dit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di \\
&= \left(\int_i \left(\frac{\sigma}{\sigma-1}\text{MC}_{Dit}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\int_i \left(\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \qquad \text{(B.4)} \\
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}
\end{aligned}
$$

B.2.4 Heterogeneous firms under monopolistic competition and IRS importing

In this section, we provide the derivations for the model where domestic manufacturers are heterogeneous in their productivity and where they can self-select into an importing technology that is subject to economies of scale. Manufacturers have access to the following technology:

$$
Y_{Dit} = \varphi_D A_{Dt} L_{Dt}^{1-\gamma} X_{Dit}^{\gamma}
$$

$$
\text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad Q_{Mit} = \left(\int_{k\in|\mathscr{L}_{it}|} {q_{Mkit}}^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}}
$$

The optimal production strategy is determined in two steps. First, conditional on the sourcing strategy $|\mathscr{L}_{it}|$, manufacturers choose the cost-minimizing bundle of labor and intermediate inputs and the cost-minimizing bundle of domestic and foreign intermediate inputs for each level of output. Second, given this production structure manufacturers determine the optimal measure $|\mathscr{L}_{it}|$ of imported intermediate input varieties subject to the fixed costs of importing.

Conditional optimal input allocation They solve a two-tiered cost minimization problem:

$$
\begin{aligned}
\min_{L_{Dit}, X_{Dt} \big| |\mathscr{L}_{it}|} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\
\text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\
& X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}(|\mathscr{L}_{it}|)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

The first-order conditions for the cost minimization problem are the following. In the upper tier, manufacturers choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xt}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\mathrm{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dt} = \gamma \mathrm{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs $(Q_{Dit}, Q_{Mit}(|\mathscr{L}_{it}|))$ given inputs prices P_{Dt} and $P_{Mit}(|\mathscr{L}_{it}|)$, both denominated in domestic currency. The first-order conditions from the second-tier problem are given by:

$$P_{Dt} Q_{Dit} = \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon-1} P_{Xit} X_{iDt} \quad \text{and} \quad P_{Mit} Q_{Mit} = (1-\omega) \left(\frac{P_{Xit}}{P_{Mit}(|\mathscr{L}_{it}|)} \right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

These first-order conditions can be combined to write the marginal cost function as:

$$\mathrm{MC}_{Dit} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xit} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) P_{Mit}(|\mathscr{L}_{it}|)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

Sourcing strategy The end problem to be solved by the manufacturing producer after solving for optimal prices and input use is to choose a measure of imported varieties. The problem is structured as follows

$$\begin{aligned}
&\max_{|\mathscr{L}_{it}|} (p_{it} - c_{it}) Y_{it} - W_t f |\mathscr{L}_{it}| \\
\text{s.t.} \qquad & c_{it} = \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \frac{1}{\varphi_i} \\
& P_{Xit} = \left[\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} |\mathscr{L}_{it}| \right]^{\frac{1}{1-\varepsilon}} \\
& Y_{it} = \left(\frac{p_{it}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt}) \\
& p_{it} = \frac{\sigma}{\sigma-1} c_{it}
\end{aligned}$$

or when all constraints are substituted in

$$\begin{aligned}
\max_{|\mathscr{L}_{it}|} \frac{1}{\sigma-1} \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} & P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \cdot \\
& \left[\frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \frac{1}{\varphi_i} \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} |\mathscr{L}_{it}| \right)^{\frac{\gamma}{1-\varepsilon}} \right]^{1-\sigma} - W_t f |\mathscr{L}_{it}|
\end{aligned}$$

Now we propose a change of variables in the maximization problem. Let

$$Z_t = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} |\mathscr{L}_{it}| \right)^{\gamma \frac{\sigma-1}{\varepsilon-1}} \Rightarrow |\mathscr{L}_{it}| = \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega {P_{Dt}}^{1-\varepsilon}}{(1-\omega) {P_{Mt}}^{1-\varepsilon}}$$

such that the maximization problem becomes

$$\max_{|\mathcal{L}_{it}|} \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt}) \left[\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma} Z_t - W_t f \frac{Z_t{}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega P_{Dt}{}^{1-\varepsilon}}{(1-\omega)P_{Mt}{}^{1-\varepsilon}}$$

The first-order condition of this problem is the following.

$$\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt}) \left[\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma} - W_t f \frac{\varepsilon-1}{\gamma(\sigma-1)} \frac{Z_t{}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}-1}}{(1-\omega)P_{Mt}^{1-\varepsilon}} = 0$$

Hence we have an expression for Z_t:

$$Z_t^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\gamma(\sigma-1)}} = \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma(\sigma-1)}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}(1-\omega)P_{Mt}{}^{1-\varepsilon}\left[\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}$$

and consequently

$$\begin{aligned}&\left(\omega P_{Dt}{}^{1-\varepsilon} + (1-\omega)P_{Mt}{}^{1-\varepsilon}|\mathcal{L}_{it}|\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}} \\ &\qquad = \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}(1-\omega)P_{Mt}^{1-\varepsilon}\left[\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}\end{aligned}$$

We can then solve for the measure of imported varieties.

$$\begin{aligned}|\mathcal{L}_{it}| = &\left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}\left(\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \varphi_i{}^{\sigma-1}\right]^{\frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}} \\ &- \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon}\end{aligned}$$

We can use this expression to determine the condition under which the measure of imported varieties is increasing in productivity

$$\frac{\partial|\mathcal{L}_i|}{\partial\varphi_i} > 0 \Leftrightarrow \frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)} > 0 \Rightarrow \gamma < \frac{\varepsilon-1}{\sigma-1}$$

and to solve for the cutoff productivity value that leads a firm to import inputs.

$$\begin{aligned}\varphi_{Mt} = &\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{f W_t}\right)^{-\frac{1}{\sigma-1}} \\ &\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}P_{Mt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}\end{aligned}$$

We can use this expression to solve for the measure of imported inputs as a function of the importing cutoff.

$$|\mathscr{L}_{it}| = \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon}\left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-1\right]$$

if $\varphi_i > \varphi_{Mt}$ and zero otherwise. We can then use this result to solve for firm-specific input prices and unit costs, respectively. We have that

$$P_{Xit} = \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{1}{1-\varepsilon}}P_{Dt}$$

if $\varphi_i \geq \varphi_{Mt}$ and $P_{Xit} = \omega^{\frac{1}{1-\varepsilon}}P_{Dt}$ when $\varphi_i < \varphi_{Mt}$.

Manufacturing price index We combine the expression for ${P_{Dt}}^{1-\sigma}$, P_{X_it} and aggregate across the firm size distribution:

$$\begin{aligned}
P_{Dt}^{1-\sigma} &= \int_i p_{it}^{1-\sigma}di = \int_i \left(\frac{\sigma}{\sigma-1}c_{it}\right)^{1-\sigma}di \\
&= \int_i \left[\frac{\sigma}{\sigma-1}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Xit}}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}di \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\int_i\left[{P_{Xit}}^{\gamma}\frac{1}{\varphi_i}\right]^{1-\sigma}di \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\left\{\int_{\underline{\varphi}}^{\varphi_{Mt}}\left[\omega^{\frac{1}{1-\varepsilon}}P_{Dt}\right]^{\gamma(1-\sigma)}\varphi_i^{\sigma-1}g(\varphi)d\varphi\right. \\
&\quad \left. + \int_{\varphi_{Mt}}^{\infty}\left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{1}{1-\varepsilon}}P_{Dt}\right]^{\gamma(1-\sigma)}\varphi_i^{\sigma-1}g(\varphi)d\varphi\right\} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\left\{\int_{\underline{\varphi}}^{\varphi_{Mt}}\varphi_i^{\sigma-1}g(\varphi)d\varphi\right. \\
&\quad \left. + \int_{\varphi_{Mt}}^{\infty}\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma(1-\sigma)}\varphi_i^{\sigma-1}g(\varphi)d\varphi\right\}
\end{aligned}$$

Now we impose that the distribution of productivities is Pareto:

$$g(\varphi) = \kappa\underline{\varphi}^{\kappa}\varphi^{-\kappa-1}$$

The first integral becomes

$$\int_{\underline{\varphi}}^{\varphi_{Mt}}\varphi^{\sigma-1}\kappa\underline{\varphi}^{\kappa}\varphi^{-\kappa-1}d\varphi = \frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\varphi^{\sigma-1-\kappa}|_{\underline{\varphi}}^{\varphi_{Mt}} = \frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\left({\varphi_{Mt}}^{\sigma-1-\kappa}-\underline{\varphi}^{\sigma-1-\kappa}\right)$$

while the second one becomes

$$\int_{\varphi_{Mt}}^{\infty}\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma(1-\sigma)}\varphi^{\sigma-1}\kappa\underline{\varphi}^{\kappa}\varphi^{-\kappa-1}d\varphi=\frac{\kappa\underline{\varphi}^{\kappa}}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\varphi_{Mt}^{\sigma-1-\kappa}$$

so that prices are

$$P_{Dt}^{1-\sigma}=\left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\cdot \left[\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}+\frac{\kappa\underline{\varphi}^{\kappa}}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right)-\underline{\varphi}^{\sigma-1-\kappa}\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\right] \tag{B.5}$$

B.3 Trade balance and labor market clearing

Combining market clearing conditions on goods markets and labor markets leads to intuitive expressions for savings and labor market clearing. These expressions depend on the assumed production and market structure in the manufacturing sector.

B.3.1 Homogeneous firms under perfect competition

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit}=X_{St}+\int_j Q_{Djt}dj, \qquad Y_{St}=C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned}Y_{Dit}=X_{Sit}+\int_j Q_{Dijt}dj&=\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}X_{St}+\int_j\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}Q_{Djt}dj\\&=\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}\left(X_{St}+\int_j Q_{Djt}dj\right)=\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt})\end{aligned}$$

where $Q_{Dt}\equiv\int_j Q_{Djt}dj$. We can also write this in aggregate form by using the corresponding aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned}Y_{Dt}\equiv\left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}&=\left(\int_i\left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}\\&=\left(\int_i P_{it}^{1-\sigma}di\right)^{\frac{\sigma}{\sigma-1}}P_{Dt}^{\sigma}(X_{St}+Q_{Dt})=X_{St}+Q_{Dt}\end{aligned}$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. Because manufacturing producers are Homogeneous, we can write the labor market clearing in terms of aggregate variables.

$$\begin{aligned} L_t = L_{St} + \int_i L_{Dit} di &= L_{St} + \int_i (1-\gamma) \frac{Y_{Dit} MC_{Dit}}{W_t} di \\ &= L_{St} + \int_i (1-\gamma) \frac{Y_{Dt} MC_{Dt}}{W_t} di = L_{St} + L_{Dt} \end{aligned}$$

Trade balance The trade balance represents the fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write it

$$\begin{aligned} TB_t &= E_t P^{\$}_{Xt} X + W_t L_t - P_t C_t = E_t P^{\$}_{Xt} X + W_t (L_{St} + L_{Dt}) - P_t C_t \\ &= E_t P^{\$}_{Xt} X + (1-\mu) P_t Y_{St} + (1-\gamma) P_{Dt} Y_{Dt} - P_t C_t \\ &= E_t P^{\$}_{Xt} X - \mu P_t C_t + (1-\gamma) P_{Dt} (Q_{Dt} + X_{St}) \end{aligned}$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs

$$\begin{aligned} Q_{Dt} &\equiv \int_i Q_{Dit} di \\ &= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \frac{P_{Xit}}{P_{Xit}} X_{Dit} di \\ &= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} Y_{Dit} di \\ &= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}} \right)^{-\sigma} (Q_{Dt} + X_{St}) \, di \\ &= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{P_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}} \right)^{-\sigma} (Q_{Dt} + X_{St}) \, di \\ &= \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon} \frac{P^{\sigma}_{Dt}}{P_{Xit}} (Q_{Dt} + X_{St}) \int_i (P_{Dit})^{1-\sigma} \, di \\ &= \frac{\gamma \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon - 1}}{1 - \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon - 1}} X_{St} \\ Q_{Dt} + X_{St} &= \frac{1}{1 - \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon - 1}} X_{St} \end{aligned}$$

Plugging this into the budget constraint yields

$$
\begin{aligned}
E_t P^{\$}_{Xt} X + W_t L_t - P_t C_t &= E_t P^{\$}_{Xt} X - \mu P_t C_t + (1-\gamma) \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_t C_t + (1-\gamma)\mu \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu \left[1 - (1-\gamma) \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} \right] P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu\gamma \frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} P_t C_t
\end{aligned}
$$

Now, we can conveniently re-write $\frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}$ using the intermediate input price index

$$
\begin{aligned}
P_{Xt}^{1-\varepsilon} &= \omega P_{Dt}^{1-\varepsilon} + (1-\omega) P_{Mt}^{1-\varepsilon} \\
\frac{1}{\omega}\left(\frac{P_{Xt}}{P_{Dt}}\right)^{1-\varepsilon} &= 1 + \frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} \\
\omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} &= \frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
\end{aligned}
$$

Then we have that

$$
\begin{aligned}
\frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}}{1-\gamma\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}} \\
&= \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}-\gamma} \\
&= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}
$$

Therefore the trade balance can be written as

$$
TB_t = E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \quad \text{where} \quad H_t \equiv \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \tag{B.6}
$$

Note that the problem for the consumer boils down to satisfying the trade balance condition in financial autarky. In addition, note that we can write the foreign intermediate input share as:

$$S_t^M \equiv \frac{P_{Mt}Q_{Mt}}{P_{Xt}X_{Dt}} = 1 - \frac{P_{Dt}Q_{Dt}}{P_{Xt}X_{Dt}} = 1 - \omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}$$

which in terms of H_t becomes:

$$\begin{aligned} H_t &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \\ \frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1} &= \frac{1-H_t}{(1-\gamma)H_t} \\ \frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{(1-\gamma)H_t}{1-H_t} \\ 1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{1-\gamma H_t}{1-H_t} \end{aligned}$$

Therefore, we have that the imported intermediate input share is given by:

$$S_t^M = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}} = \frac{\frac{(1-\gamma)H_t}{1-H_t}}{\frac{(1-\gamma\eta)H_t}{1-H_t}} = \frac{(1-\gamma)H_t}{1-\gamma H_t}$$

Labor market clearing - revisited

$$\begin{aligned} w_tL_t = w_tL_{St} + w_tL_{Dt} &= (1-\mu)PStY_{St} + (1-\gamma)P_{Dt}Y_{Dt} \\ &= (1-\mu)PStY_{St} + (1-\gamma)P_{Dt}(Q_{Dt}+X_{St}) \end{aligned}$$

Now, use the fact that $Q_{Dt} + X_{St} = \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} X_{St}$ which we can re-write in terms of H_t:

$$\begin{aligned} H_t &= \frac{1-\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}} \\ \omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} - \gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} H_t &= 1-H_t \\ \gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{\gamma(1-H_t)}{1-\gamma H_t} \\ \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\gamma H_t}{1-\gamma} \end{aligned}$$

Inserting this expression, we arrive at the labor market clearing condition

$$\begin{aligned} W_t L_t &= (1-\mu)PStY_{St} + (1-\gamma)P_{Dt}(Q_{Dt}+X_{St}) = (1-\mu)PStY_{St} + (1-\gamma)\frac{1-\gamma H_t}{1-\gamma}P_{Dt}X_{St} \\ &= (1-\mu)PStY_{St} + (1-\gamma)\frac{1-\gamma H_t}{1-\gamma}\mu P_{St}Y_{St} = \left(1-\mu+\mu-\mu\gamma H_t\right)P_{St}Y_{St} \end{aligned}$$

Using goods market clearing for final goods $Y_{St} = C_{St}$, we arrive at the labor market clearing condition:

$$W_t L_t = X_1\left(\chi_1 - \mu\gamma H_t\right)P_{St}C_{St} \qquad \text{where} \qquad X_1 = 1, \quad \chi_1 = 1 \tag{B.7}$$

In addition, note that we can write labor allocated to the service sector solely as a function of H_t as well:

$$\begin{aligned} w_t L_t &= X_1\left(\chi_1 - \mu\gamma H_t\right)P_{St}Y_{St} = X_1\left(\chi_1 - \mu\gamma H_t\right)\frac{W_t L_{st}}{1-\mu} \\ L_{St} &= \frac{1-\mu}{\chi_1 - \mu\gamma H_t}\frac{L_t}{X_1} \end{aligned}$$

B.3.2 Homogeneous firms under monopolistic competition

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit} = X_{St} + \int_j Q_{Djt}dj, \qquad Y_{St} = C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned} Y_{Dit} &= X_{Sit} + \int_j Q_{Dijt}dj = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt}dj \\ &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}\left(X_{St} + \int_j Q_{Djt}dj\right) = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt}) \end{aligned}$$

where $Q_{Dt} \equiv \int_j Q_{Djt}dj$. We can also write this in aggregate form by using the corresponding aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned} Y_{Dt} &\equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} = \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_i P_{it}^{1-\sigma}di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma}(X_{St}+Q_{Dt}) = X_{St}+Q_{Dt} \end{aligned}$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. Because manufacturing producers are Homogeneous, we can write the labor market clearing in terms of aggregate variables.

$$
\begin{aligned}
L_t = L_{St} + \int_i L_{Dit} di = L_{St} + \int_i (1-\gamma) \frac{Y_{Dit} MC_{Dit}}{W_t} di \\
= L_{St} + \int_i (1-\gamma) \frac{Y_{Dt} MC_{Dt}}{W_t} di = L_{St} + L_{Dt}
\end{aligned}
$$

Trade balance The trade balance represents the fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write it as

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X + W_t L_t - P_t C_t = E_t P^{\$}_{Xt} X + W_t L + \Pi_t - P_t C_t \\
&= E_t P^{\$}_{Xt} X + W_t (L_{St} + L_{Dt}) + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_t Y_{St} + (1-\gamma) MC_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_t Y_{St} + (1-\gamma) \frac{\sigma-1}{\sigma} P_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu P_t C_t + \left(\frac{1}{\sigma} + (1-\gamma) \frac{\sigma-1}{\sigma} \right) P_{Dt} (Q_{Dt} + X_{St})
\end{aligned}
$$

Now, we re-write $(Q_{Dt} + X_{St})$ combining the first-order condition for domestic intermediate inputs.

$$
\begin{aligned}
Q_{Dt} &\equiv \int_i Q_{Dit} di = \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \frac{P_{Xit}}{P_{Xit}} X_{Dit} di = \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} Y_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}} \right)^{-\sigma} (Q_{Dt} + X_{St}) \, di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon} \gamma \frac{\frac{\sigma-1}{\sigma} P_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}} \right)^{-\sigma} (Q_{Dt} + X_{St}) \, di \\
&= \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon} \frac{P^{\sigma}_{Dt}}{P_{Xit}} (Q_{Dt} + X_{St}) \int_i (P_{Dit})^{1-\sigma} \, di \\
&= \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon-1} (Q_{Dt} + X_{St}) \\
&= \frac{\gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon-1}}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon-1}} X_{St} \\
Q_{Dt} + X_{St} &= \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}} \right)^{\varepsilon-1}} X_{St}
\end{aligned}
$$

Plugging this is in

$$
\begin{aligned}
E_t P^{\$}_{Xt} X + W_t L_t - P_t C_t &= E_t P^{\$}_{Xt} X - \mu P_t C_t + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_t C_t + \mu\left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu \left[1 - \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}\right] P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma} \frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_t C_t
\end{aligned}
$$

Now, we can conveniently re-write $\frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}$ using the intermediate input price index

$$
P_{Xt}^{1-\varepsilon} = \omega P_{Dt}^{1-\varepsilon} + (1-\omega) P_{Mt}^{1-\varepsilon} \frac{1}{\omega}\left(\frac{P_{Xt}}{P_{Dt}}\right)^{1-\varepsilon} = 1 + \frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} \omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = \frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
$$

Then we have that

$$
\begin{aligned}
\frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} &= \frac{1-\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}} = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} - \frac{\sigma-1}{\sigma}\gamma} \\
&= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}
$$

Therefore the trade balance can be written as

$$
TB_t = E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma} H_t P_{St} C_{St} \quad \text{where} \quad H_t \equiv \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \tag{B.8}
$$

Note that the problem for the consumer boils down to satisfying the trade balance condition in financial autarky. In addition, we can write the foreign intermediate input share as:

$$
S_t^M \equiv \frac{P_{Mt}Q_{Mt}}{P_{Xt}X_{Dt}} = 1 - \frac{P_{Dt}Q_{Dt}}{P_{Xt}X_{Dt}} = 1 - \omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
$$

which in terms of H_t becomes:

$$
\begin{aligned}
H_t &= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \\
\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1} &= \frac{1-H_t}{(1-\gamma\frac{\sigma-1}{\sigma})H_t} \\
\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-H_t} \\
1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}
\end{aligned}
$$

Therefore, we have that the imported intermediate input share is given by:

$$
S_t^M = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}} = \frac{\frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-H_t}}{\frac{(1-\gamma\frac{\sigma-1}{\sigma}\eta)H_t}{1-H_t}} = \frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}
$$

Labor market clearing - revisited Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. We have:

$$
\begin{aligned}
w_t L = w_t L_{St} + w_t L_{Dt} &= (1-\mu)PStY_{St} + (1-\gamma)MC_{Dt}Y_{Dt} \\
&= (1-\mu)PStY_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}(Q_{Dt}+X_{St})
\end{aligned}
$$

Now, use the fact that $Q_{Dt}+X_{St} = \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}X_{St}$ which we can re-write in terms of H_t:

$$
\begin{aligned}
H_t &= \frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} \\
\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} - \gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} H_t &= 1-H_t \\
\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{\gamma\frac{\sigma-1}{\sigma}(1-H_t)}{1-\gamma\frac{\sigma-1}{\sigma}H_t} \\
1-\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H_t} \\
\frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}
\end{aligned}
$$

Inserting this expression, we arrive at the labor market clearing condition

$$
\begin{aligned}
W_t L &= (1-\mu) PSt Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma} P_{Dt}(Q_{Dt} + X_{St}) \\
&= (1-\mu) PSt Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}} P_{Dt} X_{St} \\
&= (1-\mu) PSt Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}} \mu P_{St} Y_{St} \\
&= \left[1-\mu+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\right] P_{St} Y_{St} \\
&= \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}-\mu\gamma(1-\gamma)\frac{\sigma-1}{\sigma}H_t\right] P_{St} Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2+\mu\frac{\sigma}{\sigma-1}-\mu\gamma H_t\right] P_{St} Y_{St}
\end{aligned}
$$

Using goods market clearing for final goods $Y_{St} = C_{St}$, we arrive at the labor market clearing condition:

$$
\begin{gathered}
W_t L = X_2\left[\chi_2 - \mu\gamma H_t\right] P_{St} C_{St} \\
\text{where} \qquad X_2 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2}{1-\gamma\frac{\sigma-1}{\sigma}}, \qquad \chi_2 \equiv (1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2+\mu\frac{\sigma}{\sigma-1}
\end{gathered}
\tag{B.9}
$$

In addition, note that we can write labor allocated to the service sector solely as a function of H_t as well:

$$
\begin{aligned}
w_t L_t &= X_2\left[\chi_2-\mu\gamma H_t\right] P_{St} C_{St} = X_2\left[\chi_2-\mu\gamma H_t\right]\frac{W_t L_{st}}{1-\mu} \\
L_{St} &= \frac{(1-\mu)}{\chi_2-\mu\gamma H_t}\frac{L_t}{X_2}
\end{aligned}
$$

B.3.3 Homogeneous firms under monopolistic competition and IRS importing

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned} Y_{Dit} = X_{Sit} + \int_j Q_{Dijt} dj &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\ &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt}) \end{aligned}$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned} Y_{Dt} \equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} &= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) = X_{St} + Q_{Dt} \end{aligned}$$

where we have used the definition of the price index.

Trade balance The trade balance represents the fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this

$$\begin{aligned} TB_t = E_t P^{\$}_{Xt} X + W_t L_t - P_t C_t &= E_t P^{\$}_{Xt} X + W_t L + \int_i \Pi_{it} di - P_t C_t \\ &= E_t P^{\$}_{Xt} X + W_t \left(L_{St} + \int_i (L_{Dit} + L_{Mit}) di\right) + \int_i \left(\frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t L_{Mit}\right) di - P_t C_t \\ &= E_t P^{\$}_{Xt} X + W_t L_{St} + W_t L_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\ &= E_t P^{\$}_{Xt} X + (1-\mu) P_t Y_{St} + (1-\gamma) MC_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\ &= E_t P^{\$}_{Xt} X + (1-\mu) P_t Y_{St} + (1-\gamma) \frac{\sigma-1}{\sigma} P_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_t C_t \\ &= E_t P^{\$}_{Xt} X - \mu P_t C_t + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) P_{Dt} (Q_{Dt} + X_{St}) \end{aligned}$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs

$$
\begin{aligned}
Q_{Dt} &= \int_j Q_{Djt}\,dj = \int_j \omega\gamma\Big(\frac{P_{Dt}}{P_{Xjt}}\Big)^{-\varepsilon}\frac{MC_{Djt}Y_{Djt}}{P_{Xjt}}\,dj \\
&= \int_j \omega\gamma\Big(\frac{P_{Dt}}{P_{Xjt}}\Big)^{-\varepsilon}\frac{MC_{Djt}}{P_{Xjt}}\left(\frac{\frac{\sigma}{\sigma-1}MC_{jt}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt})\,dj \\
&= \omega\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-\varepsilon}(X_{St}+Q_{Dt})\int_j\left(\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}P_{Xjt}^{\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\right)^{1-\sigma}P_{Xjt}^{\varepsilon-1}dj \\
&= \omega\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-\varepsilon}(X_{St}+Q_{Dt})\left(\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\right)^{1-\sigma}\int_j P_{Xjt}^{\varepsilon-1-\gamma(\sigma-1)}dj \\
&= \omega\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-\varepsilon}(X_{St}+Q_{Dt})\left(\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\right)^{1-\sigma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\sigma-1}\cdot \\
&\qquad\quad \omega^{-\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}P_{Dt}^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}P_{Dt}^{\sigma-1}(X_{St}+Q_{Dt})\left(\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\right)^{1-\sigma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\sigma-1} \\
&= \gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}(X_{St}+Q_{Dt})\left(\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\right)^{1-\sigma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\sigma-1} \\
&\qquad\quad \left[\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}\omega^{\frac{\gamma}{\varepsilon-1}}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right]^{\sigma-1} \\
&= \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}(X_{St}+Q_{Dt})
\end{aligned}
$$

Then

$$
Q_{Dt} = \frac{\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}X_{St} \quad\Rightarrow\quad Q_{Dt}+X_{St} = \frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}X_{St}
$$

Plug this back into the trade balance equation

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu P_t C_t + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) P_{Dt}(Q_{Dt} + X_{St}) \\
&= E_t P^{\$}_{Xt} X - \mu P_t C_t + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu \left[\left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} - 1\right] P_t C_t \\
&= E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma} \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_t C_t
\end{aligned}
$$

which yields the expression for the saving:

$$
TB_t = E_t P^{\$}_{Xt} X - \frac{\sigma-1}{\sigma}\mu\gamma H_t P_{St} C_{St}, \qquad H_t \equiv \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \tag{B.10}
$$

In addition, we can write the foreign intermediate input share as:

$$
\begin{aligned}
S^M_t &\equiv \frac{P_{Mt} Q_{Mt}}{P_{Xt} X_{Dt}} = 1 - \frac{P_{Dt} Q_{Dt}}{P_{Xt} X_{Dt}} = 1 - \omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} \\
&= 1 - \omega\left(\frac{P_{Dt}}{P_{Dt}\omega^{-\frac{1}{\varepsilon-1}}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}}\right)^{1-\varepsilon} = 1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}
\end{aligned}
$$

which in terms of H_t becomes:

$$
\begin{aligned}
H_t &= \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \\
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \frac{1 - H_t}{1 - \gamma\frac{\sigma-1}{\sigma} H_t} \\
1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \frac{\left(1 - \gamma\frac{\sigma-1}{\sigma}\right) H_t}{1 - \gamma\frac{\sigma-1}{\sigma} H_t}
\end{aligned}
$$

Therefore, we have that the imported intermediate input share is given by:

$$S_t^M = \frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H_t}{1-\gamma^{\frac{\sigma-1}{\sigma}} H_t}$$

Labor market clearing - revisited We start by re-writing demand for labor being used in the importing of intermediate input varieties. To this end, we rewrite profits and go back to the first-order condition for the optimal number of imported varieties. Profits can be written as:

$$\begin{aligned}\Pi_{it} &= \frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t f|\lambda_{it}| = \frac{1}{\sigma} P_{Dit} \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left[X_{St} + \int_j Q_{Djt} dj\right] - W_t f|\lambda_{it}| \\ &= \frac{1}{\sigma} P_{Dit}^{1-\sigma} \left[P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma} \left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj\right] - W_t f|\lambda_{it}| = \frac{1}{\sigma} P_{Dit}^{1-\sigma} \widetilde{Y_{Dt}} - W_t f|\lambda_{it}|\end{aligned}$$

where we have defined $\widetilde{Y_{Dt}} \equiv P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma} \left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj$. The first-order condition for the optimal number of imported varieties is given:

$$\begin{aligned}\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} - W_t f = 0 \\ \frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} \Pi_{it} = 0\end{aligned}$$

Now,

$$\begin{aligned}\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} &= (1-\sigma) \frac{\partial \ln P_{Dit}}{\partial |\lambda_{it}|} = (1-\sigma)\gamma \frac{\partial \ln P_{Xit}}{\partial |\lambda_{it}|} = (1-\sigma)\gamma \frac{\partial P_{Xit}}{\partial |\lambda_{it}|} \frac{1}{P_{Xit}} \\ &= (1-\sigma)\gamma \frac{1}{1-\varepsilon} P_{Xit}^{\frac{\varepsilon}{\varepsilon-1}} (1-\omega) P_{Mt}^{1-\varepsilon} \frac{1}{P_{Xit}} = \gamma \frac{1-\sigma}{1-\varepsilon} (1-\omega) \left(\frac{P_{Mt}}{P_{Xit}}\right)^{1-\varepsilon} \\ &= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\omega) \left(\frac{P_{Mt} |\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon} \frac{1}{|\lambda_{it}|} \\ &= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\gamma_{it}) \frac{1}{|\lambda_{it}|} \frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} = \gamma \frac{1-\sigma}{1-\varepsilon} (1-\gamma_{it}) \frac{\Pi_{it}}{|\lambda_{it}|}\end{aligned}$$

where $1-\gamma_{it} \equiv \left(\frac{P_{Mt} |\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon}$ is the domestic intermediate input share.

Going back to the first-order condition, we have:

$$
\begin{aligned}
W_t f &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|} \\
|\lambda_{it}| W_t f &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} \\
L_{Mit} W_t &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} = \frac{\gamma}{1-\varepsilon}\frac{1-\sigma}{\sigma}(1-\gamma_{it})P_{Dit}Y_{Dit} = \frac{\gamma}{\varepsilon-1}(1-\gamma_{it})MC_{Dit}Y_{Dit} \\
&= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})W_t L_{Dit} \\
L_{Mit} &= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})L_{Dit} = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)L_{Dit}
\end{aligned}
$$

where we have used the alternative expression for the domestic intermediate input share. The labor market condition becomes:

$$
\begin{aligned}
W_t L_t &= L_{St} + \int_i (L_{Dit} + L_{Mit})di = L_{St} + \int_i \left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] L_{Dit} di \\
&= L_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] P_{Dt}Y_{Dt} \\
&= (1-\mu)P_{St}C_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] P_{Dt}(X_S + Q_{Dt}) \\
&= (1-\mu)P_{St}C_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] \\
&\qquad \frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{Dt}X_{St} \\
&= (1-\mu)P_{St}Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] \\
&\qquad \frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \mu P_{St}Y_{St} \\
&= (1-\mu)P_{St}Y_{St} + \frac{\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{St}Y_{St} \\
&\qquad + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}
\end{aligned}
$$

Now re-write $\frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}$ as a function of H_t:

$$
\begin{aligned}
H_t &= \frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \\
(1-\frac{\sigma-1}{\sigma}\gamma H_t)\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= 1-H_t \\
\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \frac{\frac{\sigma-1}{\sigma}\gamma(1-H_t)}{1-\frac{\sigma-1}{\sigma}\gamma H_t} 1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \quad = \frac{1-\frac{\sigma-1}{\sigma}\gamma}{1-\frac{\sigma-1}{\sigma}\gamma H_t} \\
\frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} &= \frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}
\end{aligned}
$$

Plugging this back into the labor market clearing condition

$$
\begin{aligned}
W_tL_t &= (1-\mu)P_{St}Y_{St}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}Y_{St}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_tP_{St}Y_{St} \\
&= \left[(1-\mu)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}-\mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)}-\mu\gamma H_t\right]P_{St}Y_{St}
\end{aligned}
$$

Therefore, using goods market clearing in the services sector, we can write the labor market clearing condition:

$$
\begin{aligned}
&W_tL_t = X_3\left[\chi_3-\mu\gamma H_t\right]P_{St}C_{St} \\
&\text{where} \qquad X_3 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}, \qquad \chi_3 \equiv \frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)}
\end{aligned}
\tag{B.11}
$$

In addition, note that we can write labor allocated to the service sector solely as a function of H_t as well:

$$
\begin{aligned}
w_t L_t &= X_3 \left[\chi_3 - \mu\gamma H_t\right] P_{St} C_{St} = X_3 \left[\chi_3 - \mu\gamma H_t\right] \frac{W_t L_{st}}{1-\mu} \\
L_{St} &= \frac{(1-\mu)}{\chi_3 - \mu\gamma H_t} \frac{L_t}{X_3}
\end{aligned}
$$

B.3.4 Heterogeneous firms under monopolistic competition and IRS importing

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$
Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}
$$

Plugging in the residual demand schedules, we have

$$
\begin{aligned}
Y_{Dit} = X_{Sit} + \int_j Q_{Dijt} dj &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})
\end{aligned}
$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding aggregation for manufacturing output as dictated by the demand system:

$$
\begin{aligned}
Y_{Dt} \equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} &= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) = X_{St} + Q_{Dt}
\end{aligned}
$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic.

$$
L = L_{St} + \int_i (L_{Dit} + L_{Mit}) di
$$

Trade balance The trade balance represents the fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this in turn:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X + W_t L_t - P_{St} C_{St} = E_t P^{\$}_{Xt} X + W_t L + \int_i \Pi_{it} di - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t \left(L_{St} + \int_i (L_{Dit} + L_{Mit}) di \right) + \int_i \left(\frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t L_{Mit} \right) di - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t L_{St} + W_t L_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) MC_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) \frac{\sigma-1}{\sigma} P_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma) \frac{\sigma-1}{\sigma} \right) P_{Dt} (Q_{Dt} + X_{St})
\end{aligned}
$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs:

$$
\begin{aligned}
Q_{Dt} &= \int_j Q_{Djt} dj = \int_j \omega \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{P_{Xjt} X_{Djt}}{P_{Xjt}} dj = \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt} Y_{Djt}}{P_{Xjt}} dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{P_{jt}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt})\, dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{\frac{\sigma}{\sigma-1} MC_{jt}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt})\, dj \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \int_{\underline{\varphi}}^{\infty} \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma} P_{Xt}(\varphi)^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} P_{Xjt}^{\varepsilon-1} g(\varphi) d\varphi \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \int_{\underline{\varphi}}^{\infty} P_{Xt}(\varphi)^{\varepsilon-1-\gamma(\sigma-1)} \varphi^{\sigma-1} g(\varphi) d\varphi \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \left(\omega^{-\frac{1}{\varepsilon-1}} P_{Dt} \right)^{\varepsilon-1-\gamma(\sigma-1)} \\
&\qquad \left[\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-1} g(\varphi) d\varphi + + \int_{\varphi_{Mt}}^{\infty} \varphi^{\sigma-1} \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} g(\varphi) d\varphi \right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-1} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \\
&\qquad \left[\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-1} g(\varphi) d\varphi + \int_{\varphi_{Mt}}^{\infty} \varphi_{Mt}^{\sigma-1} g(\varphi) d\varphi \right]
\end{aligned}
$$

Now, we use the assumption that productivity is distributed according to a Pareto distribution: $g(\varphi) = \kappa \left(\underline{\varphi}\right)^{\kappa} \varphi^{-\kappa-1}$, then we have that:

$$\begin{aligned}
Q_{Dt} &= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \gamma \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_{Dt}^{\sigma-1} \left(X_{St}+Q_{Dt}\right) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\quad \kappa \left(\underline{\varphi}\right)^{\kappa} \left[\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-\kappa-2} d\varphi + \varphi_{Mt}^{\sigma-1} \int_{\varphi_{Mt}}^{\infty} \varphi^{-\kappa-1} d\varphi\right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \gamma \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_{Dt}^{\sigma-1} \left(X_{St}+Q_{Dt}\right) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\quad \kappa \left(\underline{\varphi}\right)^{\kappa} \left[\frac{1}{\sigma-1-\kappa} \left(\varphi_{Mt}^{\sigma-1-\kappa} - \underline{\varphi}^{\sigma-1-\kappa}\right) + \frac{1}{\kappa} \varphi_{Mt}^{\sigma-1-\kappa}\right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \gamma \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_{Dt}^{\sigma-1} \left(X_{St}+Q_{Dt}\right) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\quad \kappa \left(\underline{\varphi}\right)^{\kappa} \left[\varphi_{Mt}^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{\underline{\varphi}^{\sigma-1-\kappa}}{\sigma-1-\kappa}\right]
\end{aligned}$$

Now use $P_{Dt}^{1-\sigma} = \left(\frac{\sigma}{\sigma-1} \omega^{\frac{\gamma}{1-\varepsilon}} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Dt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \left[\varphi_{Mt}^{\sigma-1-\kappa} \left(\frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa} + \frac{\kappa \underline{\varphi}^{\kappa}}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right) - \underline{\varphi}^{\sigma-1-\kappa} \frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\right]$ and write:

$$\begin{aligned}
Q_{Dt} &= \gamma \frac{\sigma-1}{\sigma} \left(X_{St}+Q_{Dt}\right) \frac{\varphi_{Mt}^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{\underline{\varphi}^{\sigma-1-\kappa}}{\sigma-1-\kappa}}{\varphi_{Mt}^{\sigma-1-\kappa} \left(\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{\underline{\varphi}^{\sigma-1-\kappa}}{\kappa-(\sigma-1)}} \\
&= \gamma \frac{\sigma-1}{\sigma} \left(X_{St}+Q_{Dt}\right) \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\kappa-(\sigma-1)}} \\
&= \frac{\gamma \frac{\sigma-1}{\sigma} \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\kappa-(\sigma-1)}}}{1 - \gamma \frac{\sigma-1}{\sigma} \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\kappa-(\sigma-1)}}} X_{St} \\
Q_{Dt} + X_{St} &= \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} \left(\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa-(\sigma-1)}\right) + \frac{1}{\kappa-(\sigma-1)}}} X_{St}
\end{aligned}$$

Plugging this back into the trade balance equation:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu P_t C_t \\
&\quad + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma}\left[\frac{1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}\right] P_t C_t
\end{aligned}
$$

Therefore, we can write the trade balance equation as:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma} H_t P_{St} C_{St}, \\
\text{where} \quad H_t &\equiv \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)\right]+\frac{1}{\kappa-(\sigma-1)}}
\end{aligned}
\tag{B.12}
$$

Labor market clearing - revisited We start by re-writing demand for labor being used in the importing of intermediate input varieties. To this end, we rewrite profits and go back to the first-order condition for the optimal number of imported varieties. Profits can be written as:

$$
\begin{aligned}
\Pi_{it} &= \frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t f|\lambda_{it}| = \frac{1}{\sigma} P_{Dit}\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}\left[X_{St} + \int_j Q_{Djt}\,dj\right] - W_t f|\lambda_{it}| \\
&= \frac{1}{\sigma} P^{1-\sigma}_{Dit}\left[P^{\sigma}_{Dt} X_{St} + \int_j P^{\sigma}_{Dt}\left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj\right] - W_t f|\lambda_{it}| = \frac{1}{\sigma} P^{1-\sigma}_{Dit}\widetilde{Y_{Dt}} - W_t f|\lambda_{it}|
\end{aligned}
$$

where we have defined $\widetilde{Y_{Dt}} \equiv P^{\sigma}_{Dt} X_{St} + \int_j P^{\sigma}_{Dt}\left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj$.

The first-order condition for the optimal number of imported varieties is given:

$$
\begin{aligned}
\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} - W_t f = 0 \\
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|}\Pi_{it} = 0
\end{aligned}
$$

Now,

$$
\begin{aligned}
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} &= (1-\sigma)\frac{\partial \ln P_{Dit}}{\partial |\lambda_{it}|} = (1-\sigma)\gamma \frac{\partial \ln P_{Xit}}{\partial |\lambda_{it}|} = (1-\sigma)\gamma \frac{\partial P_{Xit}}{\partial |\lambda_{it}|}\frac{1}{P_{Xit}} \\
&= (1-\sigma)\gamma \frac{1}{1-\varepsilon} P_{Xit}^{\frac{\varepsilon}{\varepsilon-1}} (1-\omega) P_{Mt}^{1-\varepsilon} \frac{1}{P_{Xit}} = \gamma \frac{1-\sigma}{1-\varepsilon}(1-\omega)\left(\frac{P_{Mt}}{P_{Xit}}\right)^{1-\varepsilon} \\
&= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\omega)\left(\frac{P_{Mt}|\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon} \frac{1}{|\lambda_{it}|} = \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{1}{|\lambda_{it}|} \\
&= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|}
\end{aligned}
$$

where $1-\gamma_{it} \equiv \left(\frac{P_{Mt}|\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon}$ is the domestic intermediate input share. Going back to the first-order condition, we have:

$$
\begin{aligned}
W_t f &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|} \\
|\lambda_{it}| W_t f &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} \\
L_{Mit} W_t &= \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} L_{Mit} W_t = \frac{\gamma}{1-\varepsilon}\frac{1-\sigma}{\sigma}(1-\gamma_{it}) P_{Dit} Y_{Dit} \\
L_{Mit} W_t &= \frac{\gamma}{\varepsilon-1}(1-\gamma_{it}) MC_{Dit} Y_{Dit} L_{Mit} W_t = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it}) W_t L_{Dit} \\
L_{Mit} &= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it}) L_{Dit} = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right) L_{Dit}
\end{aligned}
$$

Now,

$$
\begin{aligned}
W_t L_{Mt} &\equiv \int_i W_t L_{Mit} di = \int_{\underline{\varphi}}^{\infty} W_t L_{Mt}(\varphi) g(\varphi) d\varphi = \int_{\varphi_{Mt}}^{\infty} W_t L_{Mt}(\varphi) g(\varphi) d\varphi \\
&= \int_{\varphi_{Mt}}^{\infty} \frac{\gamma}{1-\gamma} \frac{1}{\varepsilon - 1} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) L_{Dt}(\varphi) g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \frac{\sigma - 1}{\sigma} \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) P_{Dt}(\varphi) Y_{Dt}(\varphi) g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \frac{\sigma - 1}{\sigma} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) P_{Dt}(\varphi)^{1-\sigma} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) \varphi^{\sigma-1} P_{Xt}(\varphi)^{\gamma(1-\sigma)} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) \varphi^{\sigma-1} \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} \right] \varphi^{\sigma-1} g(\varphi) d\varphi
\end{aligned}
$$

Use the assumption that productivity is distributed according to a Pareto distribution: $g(\varphi) = \kappa \left(\underline{\varphi} \right)^{\kappa} \varphi^{-\kappa-1}$, then we have that:

$$
\begin{aligned}
W_t L_{Mt} &= \frac{\gamma}{\varepsilon - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \kappa \left(\underline{\varphi} \right)^{\kappa} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} \right] \varphi^{\sigma-\kappa-2} d\varphi \\
&= \frac{\gamma}{\varepsilon - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma-1} P_{Dt} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \\
&\qquad \kappa \left(\underline{\varphi} \right)^{\kappa} \varphi_{Mt}^{\sigma-1-\kappa} \left[\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa - (\sigma - 1)} \right]
\end{aligned}
$$

use again

$$P_{Dt}^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\cdot$$
$$\left[\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}+\frac{\kappa\underline{\varphi}^{\kappa}}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right)-\underline{\varphi}^{\sigma-1-\kappa}\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\right]$$

and write:

$$\begin{aligned}
W_tL_{Mt} &= \frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}P_{Dt}\left(X_{St}+Q_{Dt}\right)\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} \\
&= \frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}H_tP_{Dt}X_{St}
\end{aligned}$$

where we have used the expression for $(X_{St}+Q_{Dt})$. Now, obtain an expression for $\int_i W_tL_{Dit}di$

$$\begin{aligned}
W_tL_{Dt} \equiv \int_i W_tL_{Dit}di &= \int_{\underline{\varphi}}^{\infty} W_tL_{Dt}(\varphi)g(\varphi)d\varphi \\
&= \int_{\underline{\varphi}}^{\infty}(1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}(\varphi)Y_{Dt}(\varphi)g(\varphi)d\varphi \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}^{\sigma}\left(X_{St}+Q_{Dt}\right)\int_{\underline{\varphi}}^{\infty}P_{Dt}(\varphi)^{1-\sigma}g(\varphi)d\varphi \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}\left(X_{St}+Q_{Dt}\right)
\end{aligned}$$

Lets re-write $X_{St}+Q_{Dt}$ as a function of H_t. From the definition of H_t:

$$\begin{aligned}
&\left(1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}\right)H_t \\
&\qquad = 1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}
\end{aligned}$$

which becomes

$$
\begin{aligned}
\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} &= 1-H_t \\
1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H_t} \\
\frac{1}{\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}} \\
Q_{Dt}+X_{St} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}X_{St}
\end{aligned}
$$

Now, return to the labor market clearing condition.

$$
\begin{aligned}
W_tL_t = W_tL_{St}+\int_i(W_tL_{Dit}+W_tL_{Mit})di &= W_tL_{St}+W_tL_{Dt}+W_tL_{Mt} \\
&= (1-\mu)P_{St}Y_{St}+(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}P_{Dt}X_{St}+\frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}H_tP_{Dt}X_{St} \\
&= (1-\mu)P_{St}Y_{St}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}Y_{St}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_tP_{St}Y_{St} \\
&= \left[(1-\mu)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}-\mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2-\frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)}-\mu\gamma H_t\right]P_{St}Y_{St}
\end{aligned}
$$

Therefore, using goods market clearing in the services sector, we can write the labor market clearing condition:

$$W_t L_t = X_4 \left[\chi_4 - \mu\gamma H_t\right] P_{St} C_{St}$$

$$\text{where} \qquad X_4 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}, \qquad \chi_4 \equiv \frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)} \tag{B.13}$$

In addition, note that we can write labor allocated to the service sector solely as a function of H_t as well:

$$w_t L_t = X_4 \left[\chi_4 - \mu\gamma H_t\right] P_{St} C_{St} = X_4 \left[\chi_4 - \mu\gamma H_t\right] \frac{W_t L_{st}}{1-\mu}$$

$$L_{St} = \frac{(1-\mu)}{\chi_4 - \mu\gamma H_t}\frac{L_t}{X_4}$$

C Equilibrium

In this appendix, we prove that the equilibrium exists and is unique in all variations of the model studied in the paper. We combine the five main equations of the model, i.e., the manufacturing and service prices equations, the trade balance equation, the market clearing equation, and the endogenous openness equation, into a unique implicit equation in H only.

C.1 Perfect competition

The set of equations that determine the equilibrium is the following.

$$\begin{aligned}
P_D &= \frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left[1+\frac{1-\omega}{\omega}\left(\frac{P_D}{EP_M^{\$}}\right)^{\varepsilon-1}\right]^{-\frac{\gamma}{\varepsilon-1}}\\
P_S &= \frac{W^{1-\mu}{P_D}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}}\\
EP_X^{\$}X &= \mu\gamma H P_S C_S\\
WL &= X_1\left[\chi_1-\mu\gamma H\right]P_S C_S\\
H &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{EP_M^{\$}}{P_D}\right)^{\varepsilon-1}}
\end{aligned}$$

We start by using the H equation and the services price equation and substitute them into the manufacturing price equation to solve for P_D.

$$\begin{aligned}
P_D &= \frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S P_D^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}}\frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-\gamma H}{1-H}\right)^{\frac{\gamma}{1-\varepsilon}}\\
P_D^{\frac{1}{1-\mu}} &= \left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}
\end{aligned}$$

Second, we use trade balance, market clearing, and final goods prices and then use the H equation again

$$\begin{aligned}
EP_M^{\$} &= \frac{\mu\gamma H\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S P_D^{-\mu}\right)^{\frac{1}{1-\mu}}}{X_1\left[\chi_1-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}\\
\left(\frac{1-H}{(1-\gamma)\frac{\omega}{1-\omega}H}\right)^{\frac{1}{\varepsilon-1}}P_D^{\frac{1}{1-\mu}} &= \frac{\mu\gamma H}{1-\mu\gamma H}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\frac{LP_M^{\$}}{P_X^{\$}X}
\end{aligned}$$

Finally, we plug the expression for P_D in to find an equation in H only as follows:

$$\left(\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left((1-\gamma)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}\frac{\mu\gamma H^{\frac{\varepsilon-1}{\varepsilon}}(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}}\left(1-\gamma H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_1\left[\chi_1-\mu\gamma H\right]}=1$$

which can be written in Proposition 1 as

$$F^{\mathrm{PC}}(H,\Theta) = \Lambda_1^1(\Theta)\frac{H^{\frac{\varepsilon}{\varepsilon-1}}(1-\gamma H)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_1\left[\chi_1-\mu\gamma H\right](1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}}-1$$

$$\text{where} \qquad \Lambda^{\mathrm{PC}}(\Theta) = \mu\gamma\left(\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left((1-\gamma)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}$$

To show that at least one equilibrium exists, let $F^{\mathrm{PC}}(H,\Theta):[0,1]\to\mathbb{R}$, which is continuous on $H\in[0,1]$. Now for any $H\in[0,1]$, we have that:

$$\lim_{H\to 0}F^{\mathrm{PC}}(H,\Theta) = -1 \quad \text{and} \quad \lim_{H\to 1}F^{\mathrm{PC}}(H,\Theta) = \infty$$

then by Bolzano's Theorem, $F^{\mathrm{PC}}(H,\Theta)$ has at least one root on $H\in[0,1]$. The latter two limits follow from $H^{\frac{\varepsilon}{\varepsilon-1}}$ and $(1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}$ respectively. To show that the equilibrium is unique, consider the derivative of $F^{\mathrm{PC}}(H,\Theta)$ with respect to H:

$$\begin{aligned}\frac{\partial F^{\mathrm{PC}}(H,\Theta)}{\partial H} &= \frac{\Lambda^{\mathrm{PC}}(\Theta)}{X_1}\frac{\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{H}-\frac{\gamma}{1-\gamma}\frac{\gamma}{\varepsilon-1}\frac{1}{1-\gamma H}+\frac{\mu\gamma}{\xi_1-\mu\gamma H}+\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}\frac{1}{1-H}\right)H^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\gamma H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(\xi_1-\mu\gamma H\right)(1-H)^{\frac{1}{(\varepsilon-1)(1-\gamma)}}}\\ &= \frac{\Lambda^{\mathrm{PC}}(\Theta)}{X_1}\frac{\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{H}+\frac{\mu\gamma}{\xi_1-\mu\gamma H}+\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}\left(\frac{1}{1-H}-\frac{\gamma^2}{1-\gamma H}\right)\right)H^{\frac{\varepsilon}{\varepsilon-1}}\left(\mu-H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(\xi_1-\mu\gamma H\right)(1-H)^{\frac{1}{(\varepsilon-1)(1-\gamma)}}}>0\end{aligned}$$

Because $F^{\mathrm{PC}}(H,\Theta)$ is globally increasing in H, $F^{\mathrm{PC}}(H,\Theta)$ has only one root for $H\in[0,1)$, which ensures the uniqueness of the equilibrium.

C.2 Monopolistic competition

The set of equations that determine equilibrium in the economy with monopolistic competition is the following

$$\begin{aligned}P_D &= \frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left[1+\frac{1-\omega}{\omega}\left(\frac{P_D}{EP_M^{\$}}\right)^{\varepsilon-1}\right]^{-\frac{\gamma}{\varepsilon-1}}\\ P_S &= \frac{W^{1-\mu}P_D{}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}}\\ EP_X^{\$}X &= \mu\gamma\frac{\sigma-1}{\sigma}HP_SC_S\\ WL &= X_2\left[\chi_2-\mu\gamma H\right]P_SC_S\\ H &= \frac{1}{1+\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\left(\frac{EP_M^{\$}}{P_D}\right)^{\varepsilon-1}}\end{aligned}$$

We start by using the H equation, and the price of the final good and substitute them into the manufacturing price equation and solving it for P_D.

$$P_D = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S {P_D}^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}} \frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}$$

$$P_D^{1-\mu} = \left(\frac{\sigma-1}{\sigma}\varphi_D\right)^{-\frac{1}{1-\gamma}}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}$$

Second, we use trade balance, market clearing, and the price of the final good and then use the H equation again

$$EP_M^{\$} = \frac{\mu\gamma\frac{\sigma-1}{\sigma}H}{X_2\left[\chi_2-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S {P_D}^{-\mu}\right)^{\frac{1}{1-\mu}}$$

$$\left[\frac{1-H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\frac{1-\omega}{\omega}\right]^{\frac{1}{\varepsilon-1}} P_D^{\frac{1}{1-\mu}} = \frac{\mu\gamma\frac{\sigma-1}{\sigma}H}{X_2\left[\chi_2-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}$$

Finally, we solve for P_D to find an equation in H only as follows

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\left[\frac{1-H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}}\right]^{\frac{1}{\varepsilon-1}} = \frac{\mu\gamma\frac{\sigma-1}{\sigma}H\frac{LP_M^{\$}}{P_X^{\$}X}}{X_2\left[\chi_2-\mu\gamma H\right]}$$

which when collecting terms becomes

$$\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}$$
$$\frac{\mu\gamma\frac{\sigma-1}{\sigma}H^{\frac{\varepsilon-1}{\varepsilon}}(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_2\left[\chi_2-\mu\gamma H\right]} = 1$$

which can be written in Proposition (1) as

$$F^{\text{MC}}(H,\Theta) = \Lambda^{\text{MC}}(\Theta)\frac{H^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_2\left[\chi_2-\mu\gamma H\right](1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}} - 1$$
$$\text{where}\qquad \Lambda^{\text{MC}}(\Theta) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}\mu\gamma\frac{\sigma-1}{\sigma}$$

To show that at least one equilibrium exists, let $F^{\text{MC}}(H,\Theta) : [0,1] \to \mathbb{R}$, which is continuous on $H \in [0,1]$. Now for any $H \in [0,1]$, we have that:

$$\lim_{H\to 0} F^{\text{MC}}(H,\Theta) = -1 \quad \text{and} \quad \lim_{H\to 1} F^{\text{MC}}(H,\Theta) = \infty$$

then by Bolzano's Theorem, $F^{\text{MC}}(H,\Theta)$ has at least one root on $H \in [0,1]$. The latter two limits follow from $H^{\frac{\varepsilon}{\varepsilon-1}}$ and $(1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}$ respectively. To show that the equilibrium is unique, consider the derivative of $F^{\text{MC}}(H,\Theta)$ with respect to H:

$$
\begin{aligned}
&\frac{\partial F^{\text{MC}}(H,\Theta)}{\partial H} \\
&= \frac{\Lambda^{\text{MC}}(\Theta)}{X_2}\frac{\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{H} - \frac{\gamma}{1-\gamma}\frac{\gamma\frac{\sigma-1}{\sigma}}{\varepsilon-1}\frac{1}{1-\gamma\frac{\sigma-1}{\sigma}H} + \frac{\mu\gamma}{\xi_2-\mu\gamma H} + \frac{1}{1-\gamma}\frac{1}{\varepsilon-1}\frac{1}{1-H}\right) H^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(\xi_2-\mu\gamma H\right)(1-H)^{\frac{1}{(\varepsilon-1)(1-\gamma)}}} \\
&= \frac{\Lambda^{\text{MC}}(\Theta)}{X_2}\frac{\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{H} + \frac{\mu\gamma}{\xi_2-\mu\gamma H} + \frac{1}{1-\gamma}\frac{1}{\varepsilon-1}\left(\frac{1}{1-H} - \gamma\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\frac{\sigma-1}{\sigma}\gamma H}\right)\right) H^{\frac{\varepsilon}{\varepsilon-1}}\left(\mu-H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(\xi_2-\mu\gamma H\right)(1-H)^{\frac{1}{(\varepsilon-1)(1-\gamma)}}} > 0
\end{aligned}
$$

Because $F^{\text{MC}}(H,\Theta)$ is globally increasing in H, $F^{\text{MC}}(H,\Theta)$ has only one root for $H \in [0,1)$, which ensures the uniqueness of the equilibrium.

C.3 Increasing returns to importing

The set of equations that determine equilibrium in the economy with increasing returns to scale in importing is the following.

$$
\begin{aligned}
P_D &= \frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{\varphi_D}\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}} \\
P_S &= \frac{W^{1-\mu}P_D{}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
EP_X^{\$}X &= \mu\gamma\frac{\sigma-1}{\sigma}HP_SC_S \\
WL &= X_3\left[\chi_3-\mu\gamma H\right]P_SC_S \\
H &= \frac{1-\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \\
\frac{\varphi_M}{\varphi_D} &= \frac{1}{\varphi_D}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma}{\varepsilon-1}(1-\omega)\gamma^{\frac{\sigma-1}{\varepsilon-1}}\frac{P_D^{\sigma-1}}{Wf}P_D(X_S+Q_D)\right)^{-\frac{1}{\sigma-1}} \\
&\qquad \left(\frac{1}{A_D\Phi_D}\frac{W^{1-\gamma}P_M^{\gamma}}{(1-\gamma)^{(}1-\gamma)\gamma^{\gamma}}\right)\left[\frac{\omega}{1-\omega}\left(\frac{EP_M}{P_D}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}
\end{aligned}
$$

We use the last equation to solve for the productivity ratio as a function of H.

$$
H-\gamma\frac{\sigma-1}{\sigma}H\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = 1-\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \Rightarrow \left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = \frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}
$$

such that the price equation can be written as follows

$$P_D = \frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{\varphi_D}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}$$

leading to a similar equation as before

$$\begin{aligned}
P_D &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S P_D{}^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}}\frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}\\
P_D^{\frac{1}{1-\mu}} &= \left(\frac{\sigma-1}{\sigma}\varphi_D\right)^{-\frac{1}{1-\gamma}}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}
\end{aligned}$$

In addition, plug the first-order condition for labor use in services and the services price index into the trade balance condition:

$$EP_M^{\$} = \mu\gamma H\frac{\left((1-\mu)^{1-\mu}\mu^{\mu}P_S P_D^{-\mu}\right)^{\frac{1}{1-\mu}}}{X_3\left[\chi_3-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}$$

Use $P_D\left(X_s+Q_D\right) = \frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}}\mu P_S C_S$ and to write the cut-off equation as and then use the first-order condition for labor use in services and the services price index:

$$\begin{aligned}
\left(\frac{\Phi_M}{\Phi_D}\right) &= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_D^{\sigma-1}}{Wf}\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}}\mu P_S C_S\right)^{-\frac{1}{\sigma-1}}\left(\frac{1}{A_D\Phi_D}\frac{W^{1-\gamma}\left(EP_M^{\$}\right)^{\gamma}}{(1-\gamma)^{(}1-\gamma)\gamma^{\gamma}}\right)\\
&\quad\left[\frac{\omega}{1-\omega}\left(\frac{EP_M}{P_D}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}\\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_D^{\sigma-1}}{f}\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}}\frac{L}{X_3\left[\chi_3-\mu\gamma H\right]}\right)^{-\frac{1}{\sigma-1}}\left(\frac{\left((1-\mu)^{1-\mu}\mu^{\mu}P_s\right)^{\frac{1-\mu}{1-\gamma}}}{A_D\Phi_D(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)\\
&\quad\left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\varepsilon-1)(\sigma-1)}}P_D^{-\left(\frac{\varepsilon-1}{\sigma-1}+\frac{1-\gamma}{1-\mu}\right)}\left(EP_M^{\$}\right)^{\frac{\varepsilon-1}{\sigma-1}}
\end{aligned}$$

Plug in the expression for manufacturing prices and the cut-off as a function of H and collect terms to obtain an expression solely as a function of H:

$$\begin{aligned}
&\frac{\mu}{1-\mu}\gamma\frac{\sigma-1}{\sigma}\left(\frac{\omega}{1-\omega}(1-\gamma\frac{\sigma-1}{\sigma})\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\sigma}{\sigma-1}\frac{1}{A_D\Phi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)\left(\frac{LP_M^{\$}}{P_X^{\$}X}\right)\left(\frac{\varepsilon-1}{\gamma}\frac{1-\mu}{\mu}f\right)^{\frac{1}{\varepsilon-1}}\\
&\quad(1-\mu)^{\frac{\varepsilon-2}{\varepsilon-1}}L^{-\frac{1}{\varepsilon-1}}\frac{(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon}}H\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(X_3\left[\chi_3-\mu\gamma H\right]\right)^{\frac{\varepsilon-2}{\varepsilon-1}}} = 1
\end{aligned}$$

which can be written in Proposition (1) as

$$F^{\text{IRS}}(H,\Theta) = \frac{\Lambda^{\text{IRS}}(\Theta)\,(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon}}\,H\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(X_3\left[\chi_3-\mu\gamma H\right]\right)^{\frac{\varepsilon-2}{\varepsilon-1}}} - 1$$

$$\text{where} \qquad \Lambda^{\text{IRS}}(\Theta) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP^{\$}_{M}}{P^{\$}_{X}X}$$
$$\mu\gamma\frac{\sigma-1}{\sigma}L^{-\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon-1}{\gamma}\frac{1-\mu}{\mu}f\right)^{\frac{1}{\varepsilon-1}}(1-\mu)^{\frac{\varepsilon-2}{\varepsilon-1}}$$

To show that at least one equilibrium exists, let $F^{\text{IRS}}(H,\Theta):[0,1]\to\mathbb{R}$, which is continuous on $H\in[0,1]$. Now for any $H\in[0,1]$, we have that:

$$\lim_{H\to 0}F^{\text{IRS}}(H,\Theta) = -1 \quad \text{and} \quad \lim_{H\to 1}F^{\text{IRS}}(H,\Theta) = \infty$$

then by Bolzano's Theorem, $F^{\text{IRS}}(H,\Theta)$ has at least one root on $H\in[0,1]$. The latter two limits follow from H and $(1-H)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}$ respectively. To show that the equilibrium is unique, consider the derivative of $F^{\text{IRS}}(H,\Theta)$ with respect to H:

$$\begin{aligned}
&\frac{\partial F^{\text{IRS}}(H,\Theta)}{\partial H}\\
&= \frac{\Lambda^{\text{IRS}}(\Theta)}{X_3}\frac{\left(\frac{1}{H}-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\frac{\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}+\frac{\varepsilon-2}{\varepsilon-1}\frac{\mu\gamma}{\xi_3-\mu\gamma H}+\frac{1}{1-\gamma}\frac{1}{\varepsilon}\frac{1}{1-H}\right)H\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(\xi_3-\mu\gamma H\right)^{\frac{\varepsilon-2}{\varepsilon-1}}(1-H)^{\frac{1}{\varepsilon(1-\gamma)}}}\\
&= \frac{\Lambda^{\text{IRS}}(\Theta)}{X_3}\\
&\quad\frac{\left(\frac{1}{H}+\frac{\varepsilon-2}{\varepsilon-1}\frac{\mu\gamma}{\xi_3-\mu\gamma H}+\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}\left(\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-H}-\gamma\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\frac{\sigma-1}{\sigma}\gamma H}\right)\right)\left(\xi_3-\mu\gamma H\right)^{\frac{\varepsilon-2}{\varepsilon-1}}(1-H)^{\frac{1}{\varepsilon(1-\gamma)}}}{\left(\xi_2-\mu\gamma H\right)(1-H)^{\frac{1}{(\varepsilon-1)(1-\gamma)}}}\\
&> 0
\end{aligned}$$

The last inequality follows from rewriting:

$$\begin{aligned}
\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-H}-\gamma\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\frac{\sigma-1}{\sigma}\gamma H} &= \frac{\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)\frac{\varepsilon-1}{\varepsilon}-\frac{\sigma-1}{\sigma}\gamma^2(1-H)}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\\
&= \frac{\left(\frac{\varepsilon-1}{\varepsilon}-\frac{\sigma-1}{\sigma}\gamma^2-\frac{\varepsilon-1}{\varepsilon}\frac{\sigma-1}{\sigma}\gamma-\frac{\sigma-1}{\sigma}\gamma^2\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)-\frac{\sigma-1}{\sigma}\gamma^2(1-H)}
\end{aligned}$$

which is positive as $H\in[0,1]$ and observing that $\frac{1}{H}+\frac{\varepsilon-2}{\varepsilon-1}\frac{\mu\gamma}{\xi_3-\mu\gamma H}>0$. This is because if $\varepsilon<2$ then $\chi_3<0$. If $\varepsilon>2$ and $\varepsilon-1>\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}$, then $\chi_3>1$. If $\varepsilon-1<\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}$, then from the definition of χ_3,

$\varepsilon > 1 + \frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\frac{\mu\gamma}{\mu\gamma-\left((1-\mu)\frac{1-\frac{\sigma-1}{\sigma}\gamma}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}$, which is smaller than 1 and therefore this is always satisfied. Because $F^{\text{IRS}}(H,\Theta)$ is globally increasing in H, $F^{\text{IRS}}(H,\Theta)$ has only one root for $H \in [0,1)$, which ensures the uniqueness of the equilibrium.

D Partial equilibrium: general structure

In this section we provide the first-order linearized solutions to the non-linear equilibrium systems. We consider a first-order Taylor approximation around the steady state which we know exists and is unique in the Benchmark SOE-IRBC model, the model in which manufacturing firms compete under monopolistic competition, and the model with monopolistic competition and increasing returns to importing. In addition, we know the steady state exists and is unique in the limiting cases for $\kappa \to \infty$ and $\kappa \to \frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}$ or the heterogeneous firm model with selection and we conjecture that this remains true away from these limits.

D.1 Benchmark SOE-IRBC model

In this section, we derive the equilibrium system for the model with homogeneous producers that compete under perfect competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully rewritten in terms of H_t. In this case, only the manufacturing price index needs re-writing:

$$\begin{aligned} P_{Dt} &= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\ &= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}\left(1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \end{aligned}$$

Using the definition of H_t, we can write $1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}$ as

$$1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} = \frac{1-\gamma H_t}{1-H_t}$$

Thus, it can be re-written as:

$$P_{Dt} = \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}}$$

Given this expression for manufacturing prices, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \\
W_t L &= X_1 \left(\chi_1 - \mu\gamma H_t\right) P_{St} C_{St} \\
P_{Dt} &= \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} P^{\gamma}_{Dt}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \omega^{\frac{\gamma}{1-\varepsilon}} \left[\frac{1-\gamma H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}} \\
\mathrm{P}_{St} &= \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
H_t &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition, and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln(P_{Dt}) &= \ln\left(\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D} \frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right) - \ln(A_{Dt}) + (1-\gamma)\ln(W_t) + \gamma\ln(P_{Dt}) + \frac{\gamma}{1-\varepsilon}\ln\left(\frac{1-\gamma H_t}{1-H_t}\right) \\
p_{Dt} &= -a_{Dt} + (1-\gamma) w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma H}{1-\gamma H} + \frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= -a_{Dt} + (1-\gamma) w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t
\end{aligned}
$$

where small letters indicate percentage deviations from the steady state: $\eta_t \equiv \frac{H_t - H}{H}$. The linearized definition of H_t is given by:

$$
\begin{aligned}
\ln(H_t) &= -\ln\left[1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right] \\
\eta_t &= -(\varepsilon-1)\left[\frac{(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{P_{Mt}-P_M}{P_M} - \frac{(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{P_{Dt}-P_D}{P_D}\right] \\
\eta_t &= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right]
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St} + (1-\mu) w_t + \mu p_{Dt} \\
w_t - p_{St} &= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma)\,p_{Dt} &= -a_{Dt} + (1-\gamma)\,w_t - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt}-p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{(\varepsilon-1)(1-H)}\frac{(1-\gamma)H}{1-\gamma H}}_{\equiv \nu_{pH}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$
\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_1 - \mu\gamma H}}_{\equiv \nu_{lH}}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right) + \nu_{lH}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{pH}\eta_t\right) + \nu_{lH}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \underbrace{\left(\nu_{lH} + \frac{\mu}{1-\mu}\nu_{pH}\right)}_{\equiv \nu_{cH}}\eta_t
\end{aligned}
$$

To obtain the expenditure switching expression, we combine the relative input equation with the expression for how manufacturing prices respond to changes in openness:

$$
\begin{aligned}
\eta_t &= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{St} - \left(p_{Dt}-p_{St}\right)\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + q_t - \left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{pH}\eta_t\right)\right] \\
&= \underbrace{-\frac{(\varepsilon-1)(1-H)}{1+(\varepsilon-1)(1-H)\nu_{pH}}}_{\equiv 1/\nu_{qH}}\left[p^{\$}_{Mt} + q_t - a_{St} + \frac{1-\mu}{1-\gamma}a_{Dt}\right]
\end{aligned}
$$

D.2 Homogeneous firms under monopolistic competition

In this section, we derive the equilibrium system for the model with homogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully rewritten in terms of H_t. In this case, only the manufacturing price index needs re-writing:

$$\begin{aligned} P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\ &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}\left(1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \end{aligned}$$

Using the definition of H_t, we can write $1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}$ as

$$1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} = \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}$$

Thus, it can be re-written as:

$$P_{Dt} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}}$$

Given this expression for manufacturing prices, the non-linear goods and labor markets block is given by:

$$\begin{aligned} TB_t &= E_t P_{Xt}^{\$} X - \mu\gamma H_t P_{St} C_{St} \\ W_t L &= X_2\left(\chi_2-\mu\gamma H_t\right)P_{St}C_{St} \\ P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}} \\ \mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\ H_t &= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \end{aligned}$$

First-order linearization Linearizing the services price index, the labor market clearing condition, and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) = \ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right) - \ln(A_{Dt}) + (1-\gamma)\ln\left(W_t\right) + \gamma\ln\left(P_{Dt}\right) \\
- \frac{\gamma}{\varepsilon-1}\ln\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right] \\
p_{Dt} = -a_{Dt} + (1-\gamma)\,w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H} + \frac{H}{1-H}\right]\eta_t \\
p_{Dt} = -a_{Dt} + (1-\gamma)\,w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t
\end{aligned}
$$

where small letters indicate percentage deviations from the steady state: $\eta_t \equiv \frac{H_t-H}{H}$. The linearized definition of H_t is given by:

$$
\begin{aligned}
\ln\left(H_t\right) = -\ln\left[1 + (1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right] \\
\eta_t = -\left(\varepsilon-1\right)\left[\frac{(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_M}\left(P_{Mt}-P_M\right)\right. \\
\left. - \frac{(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_D}\left(P_{Dt}-P_D\right)\right] \\
\eta_t = -\left(\varepsilon-1\right)\left(1-H\right)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right]
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} = -a_{St} + (1-\mu)\,w_t + \mu p_{Dt} \\
0 = -a_{St} + (1-\mu)\left(w_t - p_{St}\right) + \mu\left(p_{Dt} - p_{St}\right) \\
w_t - p_{St} = \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma) p_{Dt} &= -a_{Dt} + (1-\gamma) w_t - \frac{\gamma}{\varepsilon - 1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt} - p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon - 1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)\right) - \frac{\gamma}{\varepsilon - 1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{\varepsilon-1}\frac{1}{1-H}\left[\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\right]}_{\equiv \nu_{pH}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$
\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_2 - \mu\gamma H}}_{\equiv \nu_{lH}}\eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right) + \nu_{lH}\eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{pH}\eta_t\right) + \nu_{lH}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \underbrace{\left(\nu_{lH} + \frac{\mu}{1-\mu}\nu_{pH}\right)}_{\equiv \nu_{cH}}\eta_t
\end{aligned}
$$

To obtain the expenditure switching expression, we combine the relative input equation with the expression for how manufacturing prices respond to changes in openness:

$$
\begin{aligned}
\eta_t &= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{St} - \left(p_{Dt} - p_{St}\right)\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + q_t - \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{pH}\eta_t\right)\right] \\
&= \underbrace{-\frac{(\varepsilon-1)(1-H)}{1+(\varepsilon-1)(1-H)\nu_{pH}}}_{\equiv 1/\nu_{qH}}\left[p^{\$}_{Mt} + q_t - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt}\right]
\end{aligned}
$$

D.3 Homogeneous firms under monopolistic competition and IRS Importing

In this section, we derive the equilibrium system for the model with homogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully rewritten in terms of H_t. Using the definition of H_t, we can write

$$
\begin{aligned}
H_t &= \frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \\
\left(1-\frac{\sigma-1}{\sigma}\gamma H_t\right)\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= 1-H_t \\
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}}
\end{aligned}
$$

Thus, aggregate manufacturing prices can be re-written as:

$$
\begin{aligned}
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\
&= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}}
\end{aligned}
$$

Next, we rewrite the productivity cut-off relation:

$$
\begin{aligned}
\Phi_{Mt} &= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}}\cdot \\
&\quad \frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \\
\Phi_{Mt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1}\cdot \\
&\quad \left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}
$$

Given this expression for manufacturing prices and the productivity cut-off, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \\
W_t L &= X_3 \left(\chi_3 - \mu\gamma H_t\right) P_{St} C_{St} \\
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma} P^{\gamma}_{Dt}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}} \\
\mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}} \\
\Phi^{\sigma-1}_{Mt} &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P^{\sigma-1}_{Dt}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St} C_{St}\right)^{-1} \\
&\quad \left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_t P^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition, and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) &= \ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right) + \ln\left(\frac{W^{1-\gamma}_t P^{\gamma}_{Dt}}{A_{Dt}}\right) - \frac{\gamma}{\varepsilon-1}\ln\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right] \\
p_{Dt} &= -a_{Dt} + (1-\gamma)\,w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H} + \frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= -a_{Dt} + (1-\gamma)\,w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t
\end{aligned}
$$

where small letters indicate percentage deviations from the steady state: $\eta_t \equiv \frac{H_t-H}{H}$. Solving for φ_{Mt} as a function of η_t is executed using the definition of H_t:

$$
\begin{aligned}
\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\ln\left(\frac{\Phi_{Mt}}{\Phi_D}\right) &= -\ln\left[\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}}\right] \\
\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\varphi_{Mt} &= \left(-\frac{H}{1-H} + \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t \\
\varphi_{Mt} &= -\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t
\end{aligned}
$$

Next, the linearized cut-off equation is given by:

$$
\begin{aligned}
(\sigma-1)\ln\Phi_{Mt} = \ln&\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{\mu}{f\left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{1-\sigma}}\right)^{-1}\left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}\right)\\
&-\ln\left(\frac{P_{Dt}^{\sigma-1}}{W_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}C_{St}\right)+(\sigma-1)\ln\left(\frac{1}{A_{Dt}}W_t{}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}\right)\\
&-\left(\varepsilon-1-\gamma(\sigma-1)\right)\ln\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)\\
(\sigma-1)\varphi_{Mt} =& -(\sigma-1)\,p_{Dt}+w_t+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t-c_{St}-p_{St}\\
&+(\sigma-1)(1-\gamma)\,w_t+(\sigma-1)\gamma(p_{Mt}^{\$}+e_t)-(\sigma-1)\,a_{Dt}\\
&+(\varepsilon-1-\gamma(\sigma-1))\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)\\
(\sigma-1)\varphi_{Mt} =& -(\sigma-1)\left(p_{Dt}-p_{St}-\left(1-\gamma\right)\left(w_t-p_{St}\right)-\gamma\left(p_{Mt}^{\$}+e_t-p_{St}\right)+a_{Dt}\right)\\
&-\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)
\end{aligned}
$$

Therefore, the linearized system is given by:

$$
\begin{aligned}
tb_t &= e_t+p_{Xt}^{\$}-\eta_t+p_{St}+c_{St}\\
w_t &= -\frac{\mu\gamma H}{\chi_3-\mu\gamma H}\eta_t+p_{St}+c_{St}\\
p_{Dt} &= -a_{Dt}+(1-\gamma)\,w_t+\gamma p_{Dt}-\frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t\\
p_{St} &= -a_{St}+(1-\mu)\,w_t+\mu p_{Dt}\\
\varphi_{Mt} &= -\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\\
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt}-p_{St}-\left(1-\gamma\right)\left(w_t-p_{St}\right)-\gamma\left(p_{Mt}^{\$}+e_t-p_{St}\right)+a_{Dt}\right)\\
&\quad-\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St} + (1-\mu)\,w_t + \mu p_{Dt} \\
0 &= -a_{St} + (1-\mu)\left(w_t - p_{St}\right) + \mu\left(p_{Dt} - p_{St}\right) \\
w_t - p_{St} &= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma)\,p_{Dt} &= -a_{Dt} + (1-\gamma)\,w_t - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt} - p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{\varepsilon-1}\frac{1}{1-H}\left[\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right]}_{\equiv \nu_{pH}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$
\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_3 - \mu\gamma H}}_{\equiv \nu_{lH}}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right) + \nu_{lH}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{pH}\eta_t\right) + \nu_{lH}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \underbrace{\left(\nu_{lH} + \frac{\mu}{1-\mu}\nu_{pH}\right)}_{\equiv \nu_{cH}}\eta_t
\end{aligned}
$$

To obtain the expenditure switching expression, combine the expression for how manufacturing prices respond to changes in openness and the labor market clearing condition to reduce the system:

$$
\begin{aligned}
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\right)\\
&\quad-\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&= -(\sigma-1)\Bigg(p_{Dt}-p_{St}-(1-\gamma)\left(p_{Dt}-p_{St}\right)-a_{Dt}-\frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{H}{1-H}\right]\eta_t\\
&\qquad\qquad-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\Bigg)\\
&\quad-\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&= -(\sigma-1)\gamma\left(p_{Dt}-p_{St}-\left(p^{\$}_{Mt}+e_t-p_{St}\right)\right)-\frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{H}{1-H}\right]\eta_t\\
&\quad-\left(\nu_{lH}-\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\eta_t+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&= (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)-\frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{H}{1-H}\right]\eta_t\\
&\quad-\left(\nu_{lH}-\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\eta_t
\end{aligned}
$$

Now, note that:

$$
\begin{aligned}
\frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{H}{1-H}\right]\eta_t &= \frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{pH}\eta_t\\
-\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t &= -\frac{1-\gamma}{\gamma(1-\mu)}\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)}\nu_{pH}\eta_t\\
\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\eta_t &= \frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\nu_{pH}\eta_t
\end{aligned}
$$

Then, we have that:

$$
\begin{aligned}
&(\sigma-1)\frac{1-\gamma}{\gamma(1-\mu)}\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)}\nu_{pH}\eta_t\\
&\quad=-(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)+\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{pH}\eta_t\\
&\qquad\quad+\left(\nu_{lH}-\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\nu_{pH}\right)\eta_t\\
&\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)\nu_{pH}\eta_t\\
&\quad=-(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)+\left(\nu_{lH}-\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\nu_{pH}\right)\eta_t\\
&\left(\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\nu_{pH}-\nu_{lH}\right)\eta_t=-(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)\\
&\left((\varepsilon-1)(1-H)\nu_{pH}+\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-H)\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\nu_{pH}-(1-H)\nu_{lH}\right)\eta_t\\
&\quad=-(\varepsilon-1)(1-H)\left(p^{\$}_{Mt}+e_t-p_{St}-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right)\\
&\left((\varepsilon-1)(1-H)\nu_{pH}+H-(1-H)\nu_{lH}\right)\eta_t\\
&\quad=-(\varepsilon-1)(1-H)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)
\end{aligned}
$$

where we used the expression for ν_{pH}. Therefore, we have that the expenditure switching expression becomes.

$$
\eta_t=-\frac{(1-H)(\varepsilon-1)}{H-(1-H)\nu_{lH}+(1-H)(\varepsilon-1)\nu_{pH}}\left[p^{\$}_{Mt}+q_t-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right]
$$

D.4 Heterogeneous firms under monopolistic competition and IRS importing

In this section, we derive the equilibrium system for the model with heterogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully rewritten in terms of H_t. Using the definition of H_t, we can write:

$$
\begin{aligned}
H_t &= \left[\frac{1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}\right] \\
\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right)&\left(\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}\right) \\
&= \left(\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}\right) \\
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} &= -\frac{\frac{1}{\kappa-(\sigma-1)}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t}\right)}{\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)} \\
&= -\frac{\frac{1}{\kappa-(\sigma-1)}\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)}
\end{aligned}
$$

Now define $\kappa_1 \equiv \frac{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}}{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}}$ and $\kappa_2 \equiv \frac{\frac{1}{\kappa-(\sigma-1)}}{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}}$, such that:

$$
\begin{aligned}
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} &= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\kappa_1(1-H_t)} \\
&= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}
\end{aligned}
$$

Aggregate manufacturing prices are given by

$$
\begin{aligned}
P_{Dt}^{\sigma-1} = \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}&\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\kappa\underline{\varphi}^{\kappa}} \\
&\left[\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}\right]^{-1}
\end{aligned}
$$

Using the expresion for $\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)$, we obtain:

$$\begin{aligned}
&\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}\\
&=-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)\frac{\frac{1}{\kappa-(\sigma-1)}\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)}\\
&\qquad+\frac{1}{\kappa-(\sigma-1)}\\
&=-\frac{\frac{1}{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)}\\
&=\frac{\frac{1}{\sigma-1-\kappa}\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{\kappa_1(1-H_t)+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}\\
&=\frac{\frac{1}{\sigma-1-\kappa}\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}
\end{aligned}$$

such that aggregate manufacturing prices can be written as:

$$P_{Dt}^{\sigma-1}=\left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}}\left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right)$$

Next, we rewrite the productivity cut-off relation:

$$\begin{aligned}
\Phi_{Mt}&=\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\\
&\qquad\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}\\
&=\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\\
&\qquad\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}$$

Given this expression for manufacturing prices and the productivity cut-off, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \\
W_t L &= X_4 \left(\chi_4 - \mu\gamma H_t\right) P_{St} C_{St} \\
P_{Dt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1} \frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}} \cdot \\
&\quad \left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right) \\
\mathrm{P}_{St} &= \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} &= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t} \\
\Phi_{Mt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}^{\sigma-1}}{f W_t} \frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St} C_{St}\right)^{-1} \\
&\quad \left(\frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}\left(E_t P^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1} \left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition, and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) &= \ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}}\right) + \ln\left(\frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{A_{Dt}}\right) \\
&\quad - \frac{1}{\sigma-1}\ln\left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right) \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} + \frac{1}{\sigma-1}\left(\frac{-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H} + \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t \\
&= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} + \frac{1}{\sigma-1}\left(\frac{(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H - \left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{\left(1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)}\right)\eta_t \\
&= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{1}{\sigma-1}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t
\end{aligned}
$$

Linearizing the relation between the productivity cut-off and H_t is given by:

$$
\begin{aligned}
-(\kappa-(\sigma-1))\ln\left(\frac{\Phi_{Mt}}{\underline{\varphi}}\right) &= \ln\left(\frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}\right)\\
-(\kappa-(\sigma-1))\,\varphi_{Mt} &= \left(1-\frac{(1-\kappa_1)(1-\gamma\frac{\sigma-1}{\sigma}H-H)}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t\\
\varphi_{Mt} &= -\frac{1}{\kappa-(\sigma-1)}\frac{1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}
\end{aligned}
$$

Next, the linearized cut-off equation is given by:

$$
\begin{aligned}
(\sigma-1)\ln\Phi_{Mt} = \ln&\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{\mu}{f\left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{1-\sigma}}\right)^{-1}\left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}\right)\\
&-\ln\left(\frac{P_{Dt}^{\sigma-1}}{W_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}C_{St}\right)+(\sigma-1)\ln\left(\frac{1}{A_{Dt}}W_t^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}\right)\\
&-\left(\varepsilon-1-\gamma(\sigma-1)\right)\ln\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)\\
(\sigma-1)\varphi_{Mt} = -(\sigma-1)&\,p_{Dt}+w_t+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t-c_{St}-p_{St}+(\sigma-1)(1-\gamma)\,w_t\\
&+(\sigma-1)\gamma(p^{\$}_{Mt}+e_t)-(\sigma-1)\,a_{Dt}+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)
\end{aligned}
$$

To arrive at

$$
\begin{aligned}
(\sigma-1)\varphi_{Mt} = -(\sigma-1)&\left(p_{Dt}-p_{St}-\left(1-\gamma\right)\left(w_t-p_{St}\right)-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\right)\\
&-\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St}+(1-\mu)\,w_t+\mu p_{Dt}\\
w_t-p_{St} &= \frac{1}{1-\mu}a_{St}-\frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma)\, p_{Dt} &= -a_{Dt} + (1-\gamma)\, w_t - \frac{1}{\sigma-1}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\,\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}\right)\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)\right) \\
&\quad - \frac{\gamma}{(\varepsilon-1)\,(1-H)}\,\frac{\varepsilon-1}{\gamma\,(\sigma-1)}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\,\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}\right)\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} \\
&\quad - \underbrace{\frac{1-\mu}{1-\gamma}\,\frac{\gamma}{(\varepsilon-1)\,(1-H)}\,\frac{\varepsilon-1}{\gamma\,(\sigma-1)}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}{1-\gamma^{\frac{\sigma-1}{\sigma}} H}\,\frac{\kappa_1(1-H)}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right) H}\right)}_{\equiv \nu_{pH}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$
\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_4 - \mu\gamma H}}_{\equiv \nu_{lH}}\eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{pH}\eta_t\right) + \nu_{lH}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \underbrace{\left(\nu_{lH} + \frac{\mu}{1-\mu}\nu_{pH}\right)}_{\equiv \nu_{cH}}\eta_t
\end{aligned}
$$

To obtain the expenditure switching expression, combine the expression for how manufacturing prices respond to changes in openness and the labor market clearing condition to reduce the system:

$$
\begin{aligned}
&(\sigma-1)\varphi_{Mt}\\
&\quad= -(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\right)\\
&\quad\quad -\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1))\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&\quad= -(\sigma-1)\Bigg(p_{Dt}-p_{St}-(1-\gamma)\left(p_{Dt}-p_{St}\right)-a_{Dt}\\
&\quad\quad -\frac{1}{\sigma-1}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\Bigg)\\
&\quad\quad -\left(c_{St}-\left(w_t-p_{St}\right)-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma)(\sigma-1)\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&\quad= -(\sigma-1)\gamma\left(p_{Dt}-p_{St}-\left(p^{\$}_{Mt}+e_t-p_{St}\right)\right)+\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t\\
&\quad\quad -\left(\nu_{lH}-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t+(\varepsilon-1-\gamma)(\sigma-1)\left(p^{\$}_{Mt}+e_t-p_{Dt}\right)\\
&\quad= (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)+\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t\\
&\quad\quad -\left(\nu_{lH}-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t
\end{aligned}
$$

Now, note that:

$$
\begin{aligned}
-\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\eta_t &= -\frac{1-\gamma}{\gamma(1-\mu)}\frac{\gamma(\sigma-1)}{\frac{\kappa_1}{\sigma-1}(\kappa-(\sigma-1))}\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\nu_{pH}\eta_t\\
\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t &= \frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\nu_{pH}\eta_t\\
\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t = \frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)&\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{\kappa_1}\nu_{pH}\eta_t
\end{aligned}
$$

Using these expressions, we get:

$$
\begin{aligned}
&-\frac{1-\gamma}{\gamma(1-\mu)}\frac{\gamma(\sigma-1)}{\frac{\kappa_1}{\sigma-1}\left(\kappa\left(\sigma-1\right)\right)}\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}H}{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\nu_{pH}\eta_t \\
&\qquad = (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)+\frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\nu_{pH}\eta_t \\
&\qquad\qquad -\left(\nu_{lH}-\frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{\kappa_1}\nu_{pH}\right)\eta_t
\end{aligned}
$$

Now, we use a change of variables and define ξ as the difference between κ and its smallest possible value such that the moments of the firm-size distribution still exist. Therefore, we define $\kappa=\xi\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}$. Given this definition, we can re-write κ_1 and $1-\kappa_1$ as

$$
\kappa_1=\frac{\xi\gamma(\sigma-1)}{(\xi-1)\left(\varepsilon-1\right)+\gamma(\sigma-1)},\qquad 1-\kappa_1=(\xi-1)\frac{(\varepsilon-1)-\gamma(\sigma-1)}{(\xi-1)\left(\varepsilon-1\right)+\gamma(\sigma-1)}
$$

Using these substitutions, we get:

$$
\begin{aligned}
&-\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\frac{(\varepsilon-1)-\gamma(\sigma-1)}{\xi\gamma(\sigma-1)}\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}H}{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\nu_{pH}\eta_t \\
&= (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)+\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{pH}\eta_t \\
&\left(\frac{\left(\varepsilon-1-\gamma(\sigma-1)\right)\left(1+(\xi-1)\gamma^{\frac{\sigma-1}{\sigma}}H\right)+\xi\gamma(\sigma-1)H}{(1-H)\left((1-H)\left(\varepsilon-1\right)+\gamma(\sigma-1)\right)+(\xi-1)\left(\varepsilon-1-\gamma(\sigma-1)\right)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}-\nu_{lH}\right)\eta_t \\
&= -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-\left(p_{Dt}-p_{St}\right)\right)
\end{aligned}
$$

where we used the expression for ν_{pH}. Therefore, we have that the expenditure switching expression becomes:

$$
\begin{aligned}
\eta_t &= -\frac{(1-H)(\varepsilon-1)}{\zeta(H)-(1-H)\nu_{lH}+(1-H)(\varepsilon-1)\nu_{pH}}\left[p^{\$}_{Mt}+q_t-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right] \\
\text{where}\qquad \zeta(H) &\equiv \frac{\left(\varepsilon-1-\gamma(\sigma-1)\right)\left(1+(\xi-1)\gamma^{\frac{\sigma-1}{\sigma}}H\right)+\xi\gamma(\sigma-1)H}{(1-H)\left((1-H)\left(\varepsilon-1\right)+\gamma(\sigma-1)\right)+(\xi-1)\left(\varepsilon-1-\gamma(\sigma-1)\right)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}
\end{aligned}
$$

E General equilibrium

E.1 Equilibrium process

In financial autarky, the trade balance condition implies the following equality:

$$c_{St} = e_t - p_t + p^*_{Xt} - \eta_t$$

Successively plugging in the equilibrium relations between changes in trade openness, changes in final consumption, and changes in the real exchange rate:

$$\begin{aligned}
c_{St} &= e_t - p_t + p^*_{Xt} - \eta_t \\
&= +p^*_{Xt} + a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^*_{Mt} + \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)\eta_t - \eta_t \\
&= +p^*_{Xt} + a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^*_{Mt} + \left(\nu^m_{qH}\left(H^m;\tilde{\Theta}\right) - 1\right)\eta_t \\
&= +p^*_{Xt} + a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^*_{Mt} + \frac{\left(\nu^m_{qH}\left(H^m;\tilde{\Theta}\right) - 1\right)}{\nu^m_{cH}\left(H^m;\tilde{\Theta}\right)}\left(c_{St} - a_{St} - \frac{\mu}{1-\gamma}\right)
\end{aligned}$$

Collecting terms on c_{St}, we have:

$$\begin{aligned}
&\frac{1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)}{\nu^m_{cH}\left(H^m;\tilde{\Theta}\right)} c_{St} \\
&\quad = \frac{1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)}{\nu^m_{cH}\left(H^m;\tilde{\Theta}\right)} a_{St} \\
&\qquad - \frac{(1-\mu) nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \mu(1 - nu^m_{qH}\left(H^m;\tilde{\Theta}\right))}{(1-\gamma)\nu^m_{cH}\left(H^m;\tilde{\Theta}\right)} a_{Dt} + p^*_{Xt} - p^*_{Mt} \\
c_{St} &= a_{St} + \frac{\mu\left(1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)\right) - nu^m_{cH}\left(H^m;\tilde{\Theta}\right)}{(1-\gamma)1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)} a_{Dt} \\
&\qquad + \frac{nu^m_{cH}\left(H^m;\tilde{\Theta}\right)}{1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)}\left(p^*_{Xt} - p^*_{Mt}\right)
\end{aligned}$$

Therefore, we arrive at

$$c_{St} = a_{st} + \frac{1}{1-\gamma}\left(\mu - \nu^m_c\left(H^m;\tilde{\Theta}\right)\right) a_{Dt} + \nu^m_c\left(H^m;\tilde{\Theta}\right)\left(p^*_{Xt} - p^*_{Mt}\right)$$

where $\nu^m_c\left(H^m;\tilde{\Theta}\right) \equiv \frac{nu^m_{cH}\left(H^m;\tilde{\Theta}\right)}{1+\nu^m_{cH}\left(H^m;\tilde{\Theta}\right) - \nu^m_{qH}\left(H^m;\tilde{\Theta}\right)}$. To solve for the equilibrium processes of the real exchange rate, first note that the equilibrium process of openness is given by:

$$
\begin{aligned}
\eta_t &= \frac{1}{\nu_c^m\left(H^m;\tilde{\Theta}\right)}\left(a_{St} + \frac{\mu}{1-\gamma}a_{Dt} - c_{St}\right) \\
&= \frac{1}{1-\gamma}\frac{1}{1+\nu_{cH}^m\left(H^m;\tilde{\Theta}\right)-\nu_{qH}^m\left(H^m;\tilde{\Theta}\right)}a_{Dt} - \frac{1}{1+\nu_{cH}^m\left(H^m;\tilde{\Theta}\right)-\nu_{qH}^m\left(H^m;\tilde{\Theta}\right)}\left(p_{Xt}^* - p_{Mt}^*\right)
\end{aligned}
$$

Given this, we can solve the equilibrium process for the real exchange rate:

$$
\begin{aligned}
e_t - p_t &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - p_{Mt} + \nu_{qH}^m\left(H^m;\tilde{\Theta}\right)\eta_t \\
&= a_{St} - \frac{1}{1-\gamma}\left(1-\mu-\nu_q^m\left(H^m;\tilde{\Theta}\right)\right) - \nu_q^m\left(H^m;\tilde{\Theta}\right)p_{Xt}^* - \left(1-\nu_q^m\left(H^m;\tilde{\Theta}\right)\right)p_{Mt}^*
\end{aligned}
$$

where $\nu_q^m\left(H^m;\tilde{\Theta}\right) \equiv \frac{\nu_{qH}^m\left(H^m;\tilde{\Theta}\right)}{1+\nu_{cH}^m\left(H^m;\tilde{\Theta}\right)-\nu_{qH}^m\left(H^m;\tilde{\Theta}\right)}$.

E.2 Terms-of-trade elasticity

To show that the terms-of-trade elasticity collapses to $\mu\gamma H^{\text{IRBC}}$, note that:

$$
\nu_{lH}^{\text{IRBC}} = \frac{\mu\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}
$$

and and then onto ν_{pH}^{IRBC}

$$
\nu_{pH}^{\text{IRBC}} = \frac{1-\mu}{\mu}\frac{1}{(1-H^{\text{IRBC}})(\varepsilon-1)}\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}
$$

These allow us to solve for the partial elasticity of consumption to imports

$$
\begin{aligned}
\nu_{cH}^{\text{IRBC}} &= \nu_{lH} + \frac{\mu}{1-\mu}\nu_{pH}^{\text{IRBC}} \\
&= \frac{\mu\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}} + \frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}
\end{aligned}
$$

and the partial elasticity of the RER to imports

$$
\begin{aligned}
\nu_{qH}^{\text{IRBC}} &= -\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})} - \nu_{pH}^{\text{IRBC}} = \frac{1}{((\varepsilon-1)1-H^{\text{IRBC}})}\left(1+\frac{1-\mu}{\mu}\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}\right) \\
&= -\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1-\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}
\end{aligned}
$$

such that

$$
\begin{aligned}
\nu_c^{\text{IRBC}} &= \frac{\nu_{cH}^{\text{IRBC}}}{1+\nu_{cH}^{\text{IRBC}}-\nu_{qH}^{\text{IRBC}}} \\
&= \frac{\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}\left(\frac{1-\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)\left(1-H^{\text{IRBC}}\right)}\right)}{1+\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}\left(\frac{1-\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)\left(1-H^{\text{IRBC}}\right)}\right)+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1-\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}} \\
&= \frac{\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}\left(\frac{1-\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)\left(1-H^{\text{IRBC}}\right)}\right)}{1+\frac{\mu\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1}{1-\gamma H^{\text{IRBC}}}(\mu\gamma H^{\text{IRBC}}+1-\mu\gamma H^{\text{IRBC}})} \\
&= \frac{\frac{\mu\gamma H^{\text{IRBC}}}{1-\gamma H^{\text{IRBC}}}\left(\frac{1-\gamma H^{\text{IRBC}}}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\right)}{\frac{1}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1}{1-\gamma H^{\text{IRBC}}}} \\
&= \mu\gamma H^{\text{IRBC}}\frac{\frac{1}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1}{1-\gamma H^{\text{IRBC}}}}{\frac{1}{1-\mu\gamma H^{\text{IRBC}}}+\frac{1}{(\varepsilon-1)(1-H^{\text{IRBC}})}\frac{1}{1-\gamma H^{\text{IRBC}}}} \\
&= \mu\gamma H^{\text{IRBC}}
\end{aligned}
$$

This implies that $\Xi^{\text{IRBC}}\left(H^{\text{IRBC}};\tilde{\Theta}\right)=1$.

F Quantitative excercise

F.1 Proof to proposition 3

Imports per firm We start by proving that firm-specific variety-level imports q_{Mikt} are not k specific or i specific, that is, they are the same for every importing firm.

$$
\begin{aligned}
q_{Mikt} &= \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} Q_{Mit} = \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega) \left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} X_{Dit} \\
&= \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega) \left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} \gamma \frac{MC_{it}}{P_{Xit}} Y_{it} \\
&= \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega) \left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} \gamma \frac{MC_{it}}{P_{Xit}} \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St}+Q_{Dt}) \\
&\underbrace{=}_{P_{it}\to MC_{it}} \gamma(1-\omega)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (P_{Mt})^{-\theta} \underbrace{(P_{Mit})^{\theta-\varepsilon}}_{=1} (P_{Xit})^{\varepsilon-1} (MC_{it})^{1-\sigma} (P_{Dt})^{\sigma} (X_{St}+Q_{Dt}) \\
&= \gamma(1-\omega)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (P_{Mt})^{-\theta} (P_{Xit})^{\varepsilon-1} \left(\frac{1}{A_{Dt}} \frac{1}{\varphi_i} \frac{W_t^{1-\gamma} P_{Xit}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (P_{Dt})^{\sigma} (X_{St}+Q_{Dt}) \\
&\underbrace{=}_{P_{Xit}} \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma} P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon} \left(\frac{1}{\varphi_i} \frac{W_t^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (X_{St}+Q_{Dt}) \\
&\qquad \left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right]^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma} P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon} \left(\frac{1}{\varphi_i} \frac{W_t^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (X_{St}+Q_{Dt}) \\
&\qquad \left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right]^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma} P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon+\gamma(\sigma-1)} \left(\frac{1}{\varphi_i} \frac{W_t^{1-\gamma} P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \\
&\qquad (X_{St}+Q_{Dt}) \left(\omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right)^{\varepsilon-1-\gamma(\sigma-1)} \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\sigma-1}
\end{aligned}
$$

Notice how both elements that depend on firm-level productivity cancel out, leading to

$$
\begin{aligned}
q_{Mikt} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \gamma(1-\omega)(P_{Dt})^{\sigma}(X_{St}+Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (P_{Mt})^{-1} \\
\left(\omega^{-\frac{1}{\varepsilon-1}} \frac{P_{Dt}}{P_{Mt}}\right)^{\varepsilon-1-\gamma(\sigma-1)} \varphi_{Mt}^{\sigma-1}
\end{aligned}
$$

Now recall the expression for the cutoff

$$\varphi_{Mt}^{\sigma-1} = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \left(\frac{\gamma}{\varepsilon-1}(1-\omega)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \frac{(P_{Dt})^{\sigma}(X_{St}+Q_{Dt})}{f w_t}\right)^{-1} \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1} \left(\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}$$

Notice that there are many common elements in the last two equations, leading to significant simplification

$$q_{Mikt} = (\varepsilon-1)\frac{W_t f}{P_{Mt}}$$

The total amount imported per firm in peso is then $M_{it} = (\varepsilon-1) W_t f \mathscr{L}_{it}$.

Import distribution Next, consider the closed-form solution form for the import distribution:

$$\begin{aligned}
\Pr\left(M_{it}^{\$} < M | M > 0\right) &= \Pr\left(\varphi_i < \left(\frac{1}{\varepsilon-1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon-1} + 1\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \varphi_{Mt} | \varphi_i > \varphi_{Mt}\right) \\
&= F\left(\left(\frac{1}{\varepsilon-1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon-1} + 1\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \varphi_{Mt}\right)\left(1 - F(\varphi_{Mt})\right)^{-1}
\end{aligned}$$

F.2 Proof to proposition 4

The statement is trivially true by construction in the models with a representative producer, while in the model with selection and heterogeneous firms, it follows from applying Leibniz's rule to the total amount imported per firm. Following Proposition 3, total imports can be expressed as a combination of firm-specific terms and an aggregate term as follows where $\tilde{M}_t = (\varepsilon-1) W_t f / E_t$ and such that

$$-\frac{\partial \ln M_t}{\partial \ln x_t} = -\frac{x_t}{M_t}\left[\underbrace{\int_{\varphi_{Mt}}^{\infty} \frac{\partial}{\partial x_t} \tilde{M}_t \mathscr{L}_t(\varphi) dG(\varphi)}_{\text{Intensive}} - \underbrace{\tilde{M}_t \mathscr{L}_t(\varphi_{Mt}) \frac{\partial}{\partial x_t}\varphi_{Mt}}_{\text{Extensive}}\right]$$

and the extensive margin part is zero since $\mathscr{L}_t(\varphi_{Mt}) = 0$, that is, the measure evaluated at the cutoff is nil. This is true for any shock.

F.3 Proof to proposition 5

We start with firm-level imports in ROW terms:

$$M_{it}^{*} = (\varepsilon-1)\frac{W_t f}{E_t}\mathscr{L}_{it} = \underbrace{(\varepsilon-1)\frac{W_t f}{E_t}}_{\text{Firm sub-intensive margin}} \underbrace{\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-1\right]}_{\text{Firm sub-extensive margin}}$$

Now we approximate it to the first order.

$$m_{it}^{*} = w_t - e_t + (\varepsilon-1)\left(e_t + p_{Mt}^{\$} - p_{Dt}\right) - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\frac{\left(\frac{\varphi_i}{\varphi_M}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\frac{\varphi_i}{\varphi_M}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-1}\varphi_{Mt}$$

Now we use the definition of the domestic input share:

$$\gamma_{Dit} \equiv \left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}$$

leading to

$$m_{it}^{\$} = w_t - e_t + (\varepsilon-1)\left(e_t + p_{Mt}^{\$} - p_{Dt}\right) - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\frac{\frac{1}{\gamma_{Di}}}{\frac{1}{\gamma_{Di}}-1}\varphi_{Mt}$$

Recall the linear equation for openness in the model with selection

$$\varphi_{Mt} = -\frac{1}{\kappa-(\sigma-1)}\frac{1}{1-\left[\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\tilde{\kappa}+\gamma\frac{\sigma-1}{\sigma}\right]H}\eta_t$$

We split the margins, starting with the sub-intensive

$$\begin{aligned}
w_t - e_t &= \frac{1}{1-\mu}\left(a_{St} + p_{St} - \mu p_{Dt}\right) - e_t \\
&= \frac{1}{1-\mu}\left[a_{St} + p_{St} - \mu\left(a_{St} + p_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{pH}\eta_t\right)\right] - e_t \\
&= a_{St} + p_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \frac{\mu}{1-\mu}\nu_{pH}\eta_t - e_t \\
&= a_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \frac{\mu}{1-\mu}\nu_{pH}\eta_t - q_t
\end{aligned}$$

Now recall the equation for η_t in autarky

$$\eta_t = \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + p_{Xt}^{\$} - p_{Mt}^{\$}\right)$$

and the equation for the real exchange rate

$$q_t = a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^{\$}_{Mt} + \nu_{qH} \eta_t$$

which we plug into the equation of the sub-intensive margin

$$\begin{aligned}
m^{int}_t = w_t - e_t &= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \frac{\mu}{1-\mu} \nu_{pH} \eta_t - \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^{\$}_{Mt} + \nu_{qH} \eta_t \right) \\
&= \frac{1}{1-\gamma} a_{Dt} + p^{\$}_{Mt} + \left(\frac{\mu}{1-\mu} \nu_{pH} - \nu_{qH} \right) \frac{1}{1+\nu_{cH} - \nu_{qH}} \left(-\frac{1}{1-\gamma} a_{Dt} + p^{\$}_{Xt} - p^{\$}_{Mt} \right) \\
&= \frac{1+\nu_{cH} - \nu_{qH} - \frac{\mu}{1-\mu} \nu_{pH} + \nu_{qH}}{1+\nu_{cH} - \nu_{qH}} \left(\frac{1}{1-\gamma} a_{Dt} + p^{\$}_{Mt} \right) + \frac{\frac{\mu}{1-\mu} \nu_{pH} - \nu_{qH}}{1+\nu_{cH} - \nu_{qH}} p^{\$}_{Xt} \\
&= \frac{1+\nu_{lH}}{1+\nu_{cH} - \nu_{qH}} \left(\frac{1}{1-\gamma} a_{Dt} + p^{\$}_{Mt} \right) + \frac{\frac{\mu}{1-\mu} \nu_{pH} - \nu_{qH}}{1+\nu_{cH} - \nu_{qH}} p^{\$}_{Xt}
\end{aligned}$$

and now we solve the sub-extensive margin

$$\begin{aligned}
m^{ext}_t &= (\varepsilon - 1) \left(e_t + p^{\$}_{Mt} - p_{Dt} \right) + \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \eta_t \\
&= (\varepsilon - 1) \left(e_t + p^{\$}_{Mt} - \left(a_{St} + p_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{pH} \eta_t \right) \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \eta_t \\
&= (\varepsilon - 1) \left(q_t + p^{\$}_{Mt} - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt} + \nu_{pH} \eta_t \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \eta_t \\
&= (\varepsilon - 1) \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p^{\$}_{Mt} + \nu_{qH} \eta_t + p^{\$}_{Mt} - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt} + \nu_{pH} \eta_t \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \eta_t \\
&= (\varepsilon - 1) \left(\nu_{qH} + \nu_{pH} \right) \eta_t + \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \eta_t \\
&= (\varepsilon - 1) \left(\nu_{qH} + \nu_{pH} + \frac{\sigma-1}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{\frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)}}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \right) \eta_t \\
&= (\varepsilon - 1) \left(\nu_{qH} + \nu_{pH} + \frac{\sigma-1}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{\frac{1}{1-\gamma_{Di}} \frac{1}{\kappa - (\sigma-1)}}{1 - \left[\left(1 - \gamma \frac{\sigma-1}{\sigma} \right) \tilde{\kappa} + \gamma \frac{\sigma-1}{\sigma} \right] H} \right) \cdot \\
&\quad \left[- \left(\frac{1}{1-\gamma} a_{Dt} + p^{\$}_{Mt} \right) + p^{\$}_{Xt} \right]
\end{aligned}$$

F.4 Proof of proposition 6

In this section, we explain why heterogeneity in productive efficiency and fixed costs to import are only necessary and not sufficient ingredients to obtain dynamics that are distinct from a neoclassical setting. Instead, we show that selection is a sufficient ingredient and key for generating dynamics that are different for models with and without heterogeneity in productivity.

F.5 Aggregate production function

This section derives the aggregate production function in a model without selection. It also rationalizes the choice for $X_{D,t}$ as the one that makes aggregate productivity in the model without selection equal to the degenerate productivity level in a neoclassical model defined in equation F.1. To derive the aggregate production function use the definition of Y_t

$$
\begin{aligned}
Y_t &\equiv \left(\int_i Y_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(A_{Dt}\varphi_i L_{D_i t}^{1-\gamma} X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Consider the first order condition for L_{Dit}

$$
\begin{aligned}
L_{Dit} &= (1-\gamma)\frac{\text{MC}_{it} Y_{it}}{W_t} \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{it}}{W_t}\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt}) \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{Dt}}{W_t}(X_{St}+Q_{Dt})\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma} \\
&= L_{Dt}\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma}
\end{aligned}
$$

where we have used the expression for aggregate labor demand from manufacturing for productive labor use. Insert and re-write:

$$
\begin{aligned}
Y_t &= \left(\int_i \left(A_{Dt}\varphi_i \left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} L_{Dt}^{1-\gamma} X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt} L_{Dt}^{1-\gamma}\left(\int_i \left(\varphi_i \left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt} L_{Dt}^{1-\gamma} X_{Dt}^{\gamma}\left(\int_i \left(\varphi_i \left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} \left(\frac{X_{D_i t}}{X_{Dt}}\right)^{\gamma}\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Now we obtain expression for $\frac{X_{D_i t}}{X_{Dt}}$ and $\frac{P_{it}}{P_{Dt}}$ as functions of φ only. Start by re-writing $\frac{X_{D_i t}}{X_{Dt}}$ as a function of productivity and $P_{X_i t}$

$$\begin{aligned}
\frac{X_{D_i t}}{X_{Dt}} &= \frac{X_{D_i t}}{\left(\int_i X_{D_i t}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} = \frac{\frac{\gamma \mathrm{MC}_{it} Y_{it}}{P_{X_i t}}}{\left(\int_i \left(\frac{\gamma \mathrm{MC}_{it} Y_{it}}{P_{X_i t}}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{1}{\varphi_i} P_{X_i t}^{\gamma-1} Y_{it}}{\left(\int_i \left(\frac{1}{\varphi_i} P_{X_i t}^{\gamma-1} Y_{it}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} = \frac{\frac{1}{\varphi_i} P_{X_i t}^{\gamma-1} P_{it}^{-\sigma}}{\left(\int_i \left(\frac{1}{\varphi_i} P_{X_i t}^{\gamma-1} P_{it}^{-\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\sigma-1} P_{X_i t}^{\gamma-1-\gamma\sigma}}{\left(\int_i \left(\varphi_i^{\sigma-1} P_{X_i t}^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Use the definition of $P_{X_i t}$ to write the expression as a function of φ_{Mt}

$$P_{Xit} = \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}$$

To obtain $\frac{X_{D_i t}}{X_{Dt}}$ solely as a function of φ

$$\begin{aligned}
\frac{X_{D_i t}}{X_{Dt}} &= \frac{\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}^{\gamma-1-\gamma\sigma}\right)^{\gamma-1-\gamma\sigma}}{\left(\int_i \left(\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}^{\gamma-1-\gamma\sigma}\right)^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\right)^{\gamma-1-\gamma\sigma}}{\varphi_{Mt}^{\frac{(\sigma-1)(\gamma-1-\gamma\sigma)}{\varepsilon-1-\gamma(\sigma-1)}} \left(\int_i \varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} = \frac{\varphi_i^{\frac{(\sigma-1)\varepsilon}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\int_i \varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Now re-write $\frac{P_{it}}{P_{Dt}}$ also as a function of productivity solely:

$$
\begin{aligned}
\frac{P_{it}}{P_{Dt}} &= \frac{\frac{\sigma}{\sigma-1}\mathrm{MC}_{it}}{P_{Dt}} \\
&= \frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi_i A_{Dt}}\frac{W_t^{1-\gamma}P_{X_it}^{\gamma}}{1-\gamma^{1-\gamma}\gamma^{\gamma}}}{\frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{W_t{}^{1-\gamma}P_{Dt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}\varphi_{Mt}{}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi_i A_{Dt}}\frac{W_t^{1-\gamma}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{-\frac{1}{\varepsilon-1}}P_{Dt}\right)^{\gamma}}{1-\gamma^{1-\gamma}\gamma^{\gamma}}}{\frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{W_t{}^{1-\gamma}P_{Dt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}\varphi_{Mt}{}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \frac{\frac{1}{\varphi_i}\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma}}{\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}\varphi_{Mt}{}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \varphi_i^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\sigma-1}}\underline{\varphi}^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}
\end{aligned}
$$

We can put these pieces together as:

$$
\begin{aligned}
Y_t = A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\Bigg(\int_i\Bigg\{&\varphi_i\left(\varphi_i^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\sigma-1}}\underline{\varphi}^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)^{(1-\sigma)(1-\gamma)} \\
&\left(\frac{\varphi_i^{\frac{(\sigma-1)\varepsilon}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\int_i\varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{\gamma}\Bigg\}^{\frac{\sigma-1}{\sigma}}di\Bigg)^{\frac{\sigma}{\sigma-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\gamma-1}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)(1-\gamma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left(\int_i\varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}di\right)^{\frac{\sigma}{\sigma-1}-\gamma\frac{\varepsilon}{\varepsilon-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\gamma-1}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)(1-\gamma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left(\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)^{\frac{\sigma}{\sigma-1}-\gamma\frac{\varepsilon}{\varepsilon-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\underline{\varphi}\left(\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}
\end{aligned}
$$

Note that this expression yields two insights. First, the production function in a model with heterogeneous firms, fixed costs of importing, and roundabout production, but without selection is equivalent to the production function obtained from a model with a degenerate productivity level given by equation F.1. Second, the combination of heterogeneity across firms, fixed costs of importing, and roundabout production is not sufficient to generate changes in aggregate manufacturing productivity following aggregate shocks.[27] Instead, we show in the next section that selection into importing is a sufficient condition for aggregate productivity shocks.

F.6 Model equivalence

To see this, we consider two nested specifications of the main model in which we do not allow for selection. This is implemented by assuming a minimum level of productivity that is above the importing cutoff, not only in the steady state but far enough from the cutoff that all firms in the economy are always importing ($\underline{\varphi} > \varphi_{Mt}$). To show how a model with heterogeneity and fixed costs, but without selection is dynamically equivalent to a model with only one producer, we specialize the heterogeneous firm model to a homogeneous firm model by letting $k \to \infty$ such that the productivity distribution becomes degenerate at some level φ_D. Next, we show that these two models are dynamically equivalent because they give rise to the same equilibrium conditions for the endogenous variables. Starting with the aggregate manufacturing price indices:

$$\text{Degenerate} \quad P_{Dt} = \frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left({\varphi_D}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}$$

$$\text{Pareto} \quad P_{Dt} = \frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}$$

The two latter expressions are equivalent whenever

$$\varphi_D = \underline{\varphi}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \tag{F.1}$$

and these equalities remain when we consider the other equations for these two different models. For example, in the model with degenerate heterogeneity, we have

$$H_t = \frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} = \frac{\kappa-\left(\kappa-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}\right)\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}}{\kappa-\gamma\frac{\sigma-1}{\sigma}\left(\kappa-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}\right)\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}}$$

[27] Here we refer to aggregate shocks that are not shocks to aggregate productivity in the manufacturing sector. These would trivially lead to changes in aggregate manufacturing productivity.

which are the two placeholder variables that enter the trade balance equation

$$E_t \frac{B^{\$}_{t+1}}{R_t} - E_t B^{\$}_t = E_t P^{\$}_{Xt} X - \mu\gamma \frac{\sigma-1}{\sigma} H_t P_{St} C_{St}$$

which is the same in both cases. The final check is to assess whether or not labor allocated to importing is expressed in the same equations in both cases. Under the Pareto distribution, we have

$$\begin{aligned} L_{Mt} &= f \frac{\omega}{1-\omega} \left(\frac{P_{Dt}}{E_t P_{Mt}} \right)^{1-\varepsilon} \left[\left(\frac{\underline{\varphi}}{\varphi_{Mt}} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \frac{\kappa}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1 \right] \\ &= \frac{\omega}{1-\omega} \left(\frac{P_{Dt}}{E_t P_{Mt}} \right)^{1-\varepsilon} \left[\left(\frac{\varphi_D}{\varphi_{Mt}} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1 \right] \end{aligned}$$

which again means the frameworks are in concordance.